**COMPLEXITY: Exercise No. 4**
**due in two weeks**

1. a. (Test 99)Is the following problem **NP**-complete?
   **Instance:** A graph $G = (V, E)$, a positive integer $K$.
   **Question:** Does $G$ contain an independent set of size $K$ *and* a clique of size $K$?
   b. Is the problem **NP**-complete when the $K = 100$?
   c. Is the problem **NP**-complete when the $K = |V|/2$?

2. Show that INDEPENDENT SET remains **NP**-complete even if the input graph has no clique of size 3. (Test 95)

3. Are the following problems **NP**-complete or polynomial? (prove)

   **CONNECTED DOMINATING SET**:
   **Instance:** A graph $G = (V, E)$, positive integer $K$.
   **Question:** Does $G$ contain dominating set $S$ with at most $K$ vertices such that the subgraph of $G$ induced by $S$ (i.e., the graph $G_S = (S, E \cap S \times S)$) is connected ?

   **MINIMUM LEAF SPANNING TREE**:
   **Instance:** A graph $G = (V, E)$, a positive integer $K$.
   **Question:** Is there a spanning tree for $G$ in which the number of leaves is at most $K$ ?

   **MAXIMUM LEAF SPANNING TREE**: (Test 95)
   **Instance:** A graph $G$, a positive integer $K$.
   **Question:** Is there a spanning tree for $G$ such that the number of leaves in the tree is at least $K$ ?

   **2-DOMINATING SET**: (Test 99)
   **Instance:** A graph $G = (V, E)$, a positive integer $K$.
   **Question:** Is there a set $S \subseteq V$ of size $K$ such that for any $v \in V$ there is a vertex $s \in S$ whose distance from $v$ is less or equal to 2?

   **DOMINATING PATH**: (Test 97)
   **Instance:** A graph $G = (V, E)$, positive integer $K$.
   **Question:** Does $G$ contain a simple path of length at most $K$ whose vertices form a dominating set?

   **FLOW WITH MULTIPLIERS**: (Hint: use SUBSET SUM)
   **Instance:** A directed graph $G = (V, E)$, vertices $s, t \in V$, positive integer multiplier $h(v)$ for every vertex $v \in V - \{s, t\}$, positive integer capacity $c(e)$ for every $e \in E$, and a positive integer $K$.
   **Question:** Is there a *non-negative integer* function $f$ such that:

   (a) $f(e) \leq c(e)$ for all $e \in E$.
   (b) for each $v \in V - \{s, t\}$, $\sum_{u:(u,v)\in E} h(v)f((u, v)) = \sum_{u:(v,u)\in E} f((v, u))$.
   (c) $\sum_{u:(u,t)\in E} f((u, t)) \geq K$.