COMPLEXITY: Exercise No. 5
due in two weeks

1. (Test 99)A graph $G = (V, E)$ has *bandwidth* k, if there is a bijection $f : V \to \{1, 2, \dots, |V|\}$ such that $|f(i) - f(j)| \leq k$ for each edge $(i, j) \in E$.

   a. Give a polynomial time algorithm that given a graph G with bandwidth ≤ 5 and an appropriate mapping f, it finds a maximum independent set in G.

   b. Prove or disprove: The Independent Set problem is **NP**-complete on graphs with bandwidth $\leq \sqrt{|V|}$.

2. (Test 99)Is the following problem **NP**-complete? strongly **NP**-complete?

   Instance: Non-negative integers $a_1, a_2, \dots, a_{2n}$

   Question: Is there a set $S \subseteq \{1, 2, \dots, 2n\}$ such that $|S| = n$ and $\sum_{i \in S} a_i = \sum_{i \notin S} a_i$?

3. (Test 97)

   a. Show that the following problem is **NP**-complete:

   Instance: A graph $G = (V, E)$, a positive integer weight $w(e)$ for each edge $e \in E$, and positive integers K, L.

   Question: Is there a set $S \subseteq V$ with $|S| \leq K$ such that the total weight of edges which both their endpoints are in S, is at least L ?

   b. Is the problem **NP**-complete when G is a simple path ?

4. (Test 98)Show that the following problem is in $\mathbf{NP} \cap \mathbf{coNP}$:

   Given an integer x (in binary encoding), does the number of prime divisors of x is a prime number?