COMPLEXITY: Exercise No. 6

due in two weeks

1. (Test 99)Consider the following problem:
   Input: Sets $S_1, \ldots, S_k \subseteq \mathcal{N}$ where $|S_i| = 3$ for all i.
   Goal: Find a set $I \subseteq \{1, 2, \ldots, k\}$ with maximum size such that $S_i \cap S_j = \phi$ for all $i, j \in I$.
   Give a polynomial time approximation algorithm to this problem with a constant approximation ratio.

2. (Test 99)Consider the **MINIMUM STEINER TREE** problem:
   Input: A complete graph $G = (V, E)$, a subset of the vertices $X \subseteq V$, and a length function $l(e) > 0$ defined on the edges. The lengths satisfy the triangle inequality.
   Goal: Find a tree $T = (W, F)$ such that $X \subseteq W \subseteq V$, $F \subseteq E$ and $\text{Length}(T) = \sum_{e \in F} l(e)$ is minimal.
   Give an approximation algorithm for this problem whose approximation ratio is ≤ 2.
   Note: If $X = V$ then the optimal tree is a minimum spanning tree, but this is not true if $X \subset V$.

3. Give an approximation algorithm for **MAXIMUM CLIQUE** whose inverse ratio is $O(n/\log n)$ where n is the number of vertices in the input graph (i.e., there is a constant c such that $A(I) \geq \frac{c \log n}{n} OPT(I)$ for any instance I).

4. Consider the following greedy algorithm A for **MAX KNAPSACK**:

   (a) Reorder items so that $v_1/s_1 \geq v_2/s_2 \geq \cdots \geq v_n/s_n$.

   (b) for $i = 1$ to n do: insert item i into the knapsack if it still fits in the remaining space.

   (c) if the largest value of a single item is larger than the value of the solution obtained in (b), take that single item as the solution instead.

   Prove that $R_A = \frac{1}{2}$.

5. Prove that the following generalization of *Minimum Circuit* is also in $\Pi_2\mathbf{P}$:
   Instance: A Boolean circuit C, and integer k
   Question: Are there no more than k circuits, each with fewer gates than C, computing the same function ? (i.e. C is one of the $k+1$ minimum circuits)