Algorithmic Methods	**Fall Semester, 2014/15**

Final Exam: February 9, 2015

Lecturer: Prof. Yossi Azar

Solve **4 out of the 5** questions. Write short but full and accurate answers. Each question should start on a new page and each of its parts should not exceed a page. No extra material is allowed.

1. We are given a strongly connected graph $G = (V, E)$ ($|V| = n$ is even) with a non-negative weights w_e for $e \in E$ and also two vertices $x, y \in V$. Increasing the weight of an edge e by each unit costs c_e for each $e \in E$. One need to increase the weights of the edges with minimum total cost such that the length of any (not necessary simple) path from x (to anywhere) with exactly $n/2$ edges is at most twice the length of any (not necessary simple) path from y (to anywhere) with exactly $n/2$ edges.
   (a) (20 points) Form an LP for the problem and show how to solve it by a polynomial time algorithm.
   (b) (5 points) What if we need that the length any path from x of *at most* $n/2$ edges is at most twice the length of any path from y with *at least* $n/2$ edges.

2. Consider an approximation algorithm for MAX-SAT which is based on LP and randomized rounding. Let x be the LP optimal fractional solution.
   (a) (2 points) Show that $(0.8 - 0.6/k)^k \le 1/4$ for any integer $k \ge 1$ (you can check for $k \le 6$).
   (b) (18 points) Show that if we randomly round each variable independently according to the function $p_i = 0.2 + 0.6x_i$ then we get a 3/4-approximation for MAX-SAT.
   (c) (5 points) Consider the formula with two clauses $x_1 \vee \bar{x}_2$ and $\bar{x}_1 \vee x_2$. Characterize all optimal fraction solutions. Determine the one that causes the worst expected approximation using the rounding above.

3. We are given m unrelated machines and $2n$ jobs partitioned into n pairs. We need to choose one job from each pair and assign it to the machines. The goal is to minimize the maximum load.
   (a) (12 points) Assume the load of a job can be split among the machines. Design a 2 approximation algorithm against an optimum which also can split the load of the job it chooses from each pair.
   (b) (13 points) Assume a job has to be assigned to a single machine. Design a 3 approximation algorithm against an optimum which also needs to assign a job it chooses from each pair to a single machine.

4. We are given a tree and requests (s_i, t_i) with a bandwidth w_i and a value v_i for $1 \le i \le n$. The goal is to maximize the total value of requests in a feasible subset.
   (a) (15 points) Design a 3 approximation algorithm where a feasible set is a subset of the requests with total bandwidth at most 1 on each vertex.
   (b) (10 points) Modify the algorithm and the proof to design a 6 approximation algorithm where a feasible set is a subset of the requests with bandwidth at most 1 on each edge.

5. We are given an undirected connected graph $G = (V, E)$ with a non-negative capacities c_e for each edge $e \in E$. You need to pack (fractionally) spanning tree, i.e. assign each tree a non-negative weight such that the total load on any edge does not exceed its capacity. The goal is to maximize the sum of the weights of the packed spanning trees.
   (a) (5 points) Show that there exists an optimal solution in which at most $|E|$ spanning trees get non-zero weights (Hint: you may write LP with non-polynomial number of variables).
   (b) (10 points) Show how to find the value of the optimal solution in polynomial time. (Hint: dual - you do not need to find the trees but only the optimal value).
   (c) (10 points) Assume we can add (fractional) capacities to the edges such that the total addition to all edges is at most 2. Find the new value of the optimal solution of the packing in polynomial time.

The duration of the exam is 3 hours. GOOD LUCK