Riesz Representation Theorem - Compact case

Let X be compact Hausdorff, $C(X)$ the continuous functions on f (with sup-norm). A continuous linear operator on $C(X)^*$ is a linear map $L: C(X) \to \mathbb{C}$ s.t. $\sup_{\substack{f \in C(X) \\ \|f\|_\infty = 1}} |L(f)| < \infty$.

L is positive if $f \geq 0 \Rightarrow L(f) \geq 0$.
A Borel measure μ on X is regular if $\mu(X) < \infty$ and
(*) For every Borel set E, $\mu(E) = \inf\{\mu(U) : E \subset U, U \text{ open}\}$
(**) " " " " " $\mu(E) = \sup\{\mu(K) : K \subset E, K \text{ compact}\}$.

Riesz Rep'n theorem The map $\mu \mapsto L_\mu(f) = \int_X f \, d\mu$

is a bijection {regular Borel measures on X}

$$\updownarrow$$

(***)

{positive continuous linear operators on $C(X)$}

Now suppose X is locally compact Hausdorff.
Let $C_c(X)$ denote continuous functions on X of compact support. A measure μ on X is Radon if it satisfies (*), (**) above, and if $\mu(K) < \infty$ for any compact K.

Riesz Rep'n theorem, general case: For any positive continuous linear functional $L : C_c(X) \to \mathbb{C}$, there is a Radon measure μ on X s.t. $L = L_\mu$, and μ is uniquely determined by L.

Remark: This means that in (***) there is an injective map $\uparrow$.
It need not be surjective (some μ do not correspond to cts. functionals).