**Topics in Analysis on Manifolds**

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**1.** Notion of a smooth manifold (in $\mathbf{R}^N$). Morphisms. Submanifolds
**2.** Notion of tangent and cotangent spaces. Differentials.
**3.** Submersive morphisms. Immersions.
**4.** Families of morphisms
**5.** Notion of transversality.

Let $\nu : X \to Y$ and $\lambda : Z \to Y$ be morphisms of manifolds. We say that $\nu$ and $\lambda$ are **transversal** if for any $x \in X, z \in Z$ either $\nu(x) \neq \lambda(z)$ or they are equal and $d\nu(T_x X) + d\lambda(T_z Z) = T_y Y$ , where $y = \nu(x) = \lambda(z)$.

**6.** First main result: **Sard's Lemma.**

Let $\nu : X \to Y$ be a morphism of smooth manifolds. Then for almost every point $y \in Y$ $\nu$ is transversal to the imbedding $i : y \to Y$.

**7.** Corollary. Let $\nu : S \times X \to Y$ be a family of morphisms and $\lambda : Z \to Y$ a morphism.

Suppose we know that the morphism $\nu$ is transversal to $\lambda$.

Then for almost every $s \in S$ the morphism $\nu_s : X \to Y$ is transversal to $\lambda$.

**8.** Second main result. **Deformation theorem.**

Let $\nu_0 : X \to Y$ me a morphism of manifolds. Then there exists a base $S$ and a family of morphisms $\nu : S \times X \to Y$ such that

(i) $\nu$ is a submersive morphism ( in particular it is transversal to any morphism $\lambda : Z \to Y$).

(ii) For some point $s_0 \in S$ the morphism $\nu_{s_0}$ coincides with $\nu_0$.

**9.** Corollary **Mooving Lemma.**

For any morphism $\nu : X \to Y$ there exists an arbitrary close deformation $\nu' : X \to Y$ which is transversal to a given morphism $\lambda : Z \to Y$.