Problem assignment 1

Representations of p-adic groups

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We will denote by $\mathbf{F}$ a fixed local non-Archimedian field.

1. Let V be a finite dimensional vector space over $\mathbf{F}$ considered as an l-group. Denote by $\hat{V}$ the Pontryagin dual group $\hat{V} = Hom(V, \mathbf{C}^*)$.

(i) Show that there exists a natural (though not canonical) isomorphism $\hat{V} \cong V^*$.

(ii) Show that the Hecke algebra $\mathcal{H}(V)$ is naturally isomorphic to the algebra $S(\hat{V})$.

(iii) Show that the category $\mathcal{M}(V)$ is naturally equivalent to the category $Sh(\hat{V})$.

2. Let $G = GL(2, \mathbf{F})$ be the group of 2×2 matrices. Denote by P the subgroup of matrices $g = (a_{ij}) \in G$ for which $a_{21} = 0$ and $a_{22} = 1$.

(i) Show that P has a normal subgroup V isomorphic to the additive group $\mathbf{F}$.

(ii) Show that the quotient group P/V is naturally isomorphic to the multiplicative group $\mathbf{F}^*$ and that this group acts on V via multiplication.

3. Show that the category of smooth representations $\mathcal{M}(P)$ is equivalent to the category $Sh_{\mathbf{F}^*}(\hat{V})$ of $\mathbf{F}^*$-equivariant sheaves on the l-space $\hat{V}$.

4. Using problem 3 give classification of all irreducible (smooth) representations of the group P.