Problem assignment 2

Algebraic Geometry and Commutative Algebra II

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In this assignment X is a smooth projective curve of genus g and D a divisor on X. We define $degD$ and $l(D)$ like in class and define $h(D)$ from the formula $l(D) - h(D) = degD + 1 - g$.

By definition, $h(D) \geq 0$ and for some divisors $h(D) = 0$.

1. Suppose we know that $l(D) \neq 0$. Show that for almost every point $x \in X$ we have $l(D - x) = l(D) - 1$.

2. Show the following properties of $h(D)$:

(i) If $D \approx D'$ then $h(D) = h(D')$.

(ii) For every point $x \in X$ we have $h(D) \geq h(D + x) \geq h(D) - 1$.

3. Show that if $degD \geq g$ then D is equivalent to an effective (i.e. positive) divisor.

4. Show that if $degD \geq 2g - 1$ then $h(D) = 0$ i.e. $l(D) = degD + 1 - g$.

5. Let P be a point on X. Show that the variety $X \setminus P$ is affine.

6. Fix n distinct points $x_1, ..., x_n \in X$. A collection of these points and a collection of rational functions $F = (f_1, ..., f_n)$ we call **Cousin data**.

We say that a rational function f is comparable with Cousin data F if for every point x_i the functions f and f_i have the same polar part at x_i (i.e. $f - f_i$ is regular at x_i).

Fix one more point $y \in X$ distinct from all x_i and a number n. We would like to solve the following Cousin problem:

Find a rational function $f \in k(X)$ which is comparable with Cousin data F, regular outside points $x_1, ..., x_n, t$ and has pole of order $\leq n$ at y.

Show that if n is sufficiently large this Cousin problem always could be solved.

Give some estimate on the minimal value of n when you could guarantee that Cousin problem has a solution.