Problem assignment 5

Algebraic Geometry and Commutative Algebra II

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1. Let $F : \mathcal{A} \to \mathcal{B}$ be an additive functor between abelian categories. Suppose it maps any SES (short exact sequence) into a left SES. Show that it is left exact, i.e. it maps left SES into left SES.

2. Let X be a projective variety, L an invertible $\mathcal{O}$-module on X. Show that the following conditions are equivalent:

(i) L is ample

(ii) For any coherent $\mathcal{O}_X$-module F for large k the twisted module $F(k) := F \otimes L^{\otimes k}$ is acyclic.

(iii) For any variety S and any coherent $\mathcal{O}$-module F on $S \times X$ for large k the twisted $\mathcal{O}$-module $F(k)$ is p_* acyclics, where $p : S \times X \to X$ is the projection.

3. Let X be a projective variety with an ample invertible sheaf L. Let N be some invertible sheaf on X. Show that for large k the sheaf $N(k) := N \otimes L^{\otimes k}$ is ample (and even very ample).

4. Let X be an algebraic variety, F coherent sheaf on X. Show that for a point $x \in X$ the following conditions are equivalent

(i) F is free near point x

(ii) $Tor_1(F, \delta_x) = 0$

5. Let $0 \to L \to M \to N \to 0$ be a SES of modules. Show that if M and N are flat then also L i s flat.

6. Let C be a complex of A-modules. Suppose we know that it is exact, bounded above and consists of flat modules.

Show that for any A-module J the complex $C_J := C \otimes_A J$ is exact.