Problem assignment 3

Analysis on Manifolds

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1. Consider a morphism of manifolds $\nu : X \to Y$ and a submanifold $Z \subset Y$.

Let us fix a morphism $p : Y \to S$ where S is some manifold (which we would like to consider as a base of some families of manifolds and morphisms). Then over every point $s \in S$ we can consider fibers X_s, Y_s, Z_s and the morphism $\nu_s : X_s \to Y_s$.

Suppose we know that the morphism ν is transversal to the submanifold Z.

Show that for almost every point $s \in S$ the fibers are manifolds and the morphism ν_s is transversal to the submanifold Z_s.

2. Show that any morphism $\nu : S^8 \to S^3 \times S^5$ has degree 0.

3.. Let $\nu : X \to Y$ be a morphism of oriented manifolds of the same dimension n. Suppose we know that X is compact and Y is connected but not compact.

Show that ν has degree 0.

4. Let $(c, M), (e, T)$ be two cycles in a manifold X of complementary dimension with intersection index $int(c, e) = i$. Let us assume that the manifold M is connected and consider a morphism of manifolds of the same dimension $\nu : N \to M$ (all manifolds are assumed to be compact and oriented). Then we have a new cycle $(c' = c \circ \nu, N)$ in X.

Compute the intersection index $int(c', e)$ if $\deg \nu = d$.

5. Let $(c, M), (e, T)$ be two **continuous** cycles of complementary dimension in a manifold X. Suppose we know that near the set $W = M \times_X T$ both morphisms c, e are smooth and transversal.

How to compute the intersection index $int(c, e)$?

6. Let $(c, M), (e, T)$ be two cycles in a manifold X of complementary dimension.

Show that the intersection index $int(c, e)$ will not change if we replace the cycle (c, M) by a cobordant cycle (c', N).

This means that there exists a manifold with boundary R and a morphism $\nu : R \to X$ such that the boundary ∂R is isomorphic to $M \coprod N$, orientation on μ_R induces orientations μ_M and $-\mu_N$ on the boundary and the restriction of ν to the boundary coincides with $c \coprod c'$.

7. (i) Let M be a non-empty compact closed manifold of dimension $n > 0$. Show that M is not contractable.

(ii) Suppose M is a boundary of some manifold W. Show that then there is no continuous retraction $p : W \to M$.