Topics for final exam

Introduction to Differential Geometry.

Joseph Bernstein Fall 2006.

Here are the topics which can appear on the final exam.

1. Linear algebra: vector spaces, basis, dimension, coordinates, dual space, quadratic forms

2. Geometry of an Euclidean spaces. Diagonalization of a quadratic form in orthonormal basis.

3. Metric spaces, topology on metric spaces, continuity. The notion of compactness. Main properties and applications.

4. Differentiable functions on open subsets of vector spaces. Partial derivatives. Algebra of smooth functions. Inverse function theorem.

5. Smooth domains and abstract domains. Coordinate systems.
Notion of a k-dimensional submanifold of a domain.

6. Tangent space $T_a(X)$ to the domain X. Equivalence of geometric and algebraic definitions. Basis of the tangent space and of the cotangent apace.

7. Differential of a morphism of domains. Differential of a function.

8. Vector fields - geometric and algebraic descriptions. Commutator of vector fields.
Vector fields as infinitesimal diffeomorphisms. Lie derivative of geometric objects with respect to a vector field.

9. Differential forms, DeRham differential.
Weyl calculus of differential forms.

10. Partition of unity and its application to construction of vector fields, functions, differential forms and so on.

11. Orientation. Integration of top degree differential forms over an oriented manifold. Properties of integration.

12. Manifolds with boundary. Stokes theorem.

13. **Applications.** Integration over cycles. Conditions of equality of integrals over homotopic or bordant cycles.
Notion of degree, Brauer theorems, rotation number and so on.

14. Geometry of curves in $\mathbf{R}^2$ and $\mathbf{R}^3$. Natural parameter, Frenet frames and Frenet formulas. Computation of curvature and torsion.

15. Geometry of surfaces in $\mathbf{R}^3$. First fundamental form and inner geometry. Second fundamental form. Relation to computation of curvatures of curves on the surface.

16. Principal curvatures of a surface. Gauss curvature and mean curvature. Computation of first and second fundamental forms. Computation of Gauss curvature and mean curvature.

17. Notion of a Riemannian manifold. Identification of vector fields and covector fields on a Riemannian manifold. Gradient of a function.
Application to construction of vector fields on a manifold.

18. Statement of Gauss egregium theorem.

Topics discussed at the end of the course which will not appear on the exam.

19. Calculus of differential forms $\Omega(M, V)$ on a manifold M with values in a vector space V,
Method of moving frames. Fundamental equations of moving frames.
Darbaux frames and the proof of the Gauss egregium theorem.

20. Notion of a vector bundle. Connection on a vector bundle. Curvature of a connection.

21. Affine connections. Torsion of an affine connection.
Levi-Cevita affine connection on a Riemannian manifold.
Curvature of a Riemannian manifold.