

## Representations of Finite Groups.

### Syllabus.

- 1.** Groups.
  - (i) Basic definitions and properties.
  - (ii) Actions of groups on sets. Category of  $G$ -sets. Natural constructions with  $G$ -sets.
  - (iii) Counting principle and applications.
- 2.** Representations of a finite group  $G$ .
  - (i) Basic definitions. Category of representations of a group  $G$ .
  - (ii) Irreducible representations. Schur's lemmas.
  - (iii) Natural constructions with representations.
  - (iv) Complete reducibility. Application to the description of endomorphism algebras.
- 3.** Basic results about representations of finite groups.
  - (i) Intertwining numbers and their properties.
  - (ii) Decomposition of the regular representation.
  - (iii) Group algebra and its structure.
  - (iii) Burnside theorem and its corollaries.
- 4.** Character theory.
  - (i) Definition of a character.
  - (ii) Orthogonality relations. Character rings.
  - (iii) Brauer's theorem
- 5.** Frobenius reciprocity and Mackey theory.
  - (i) General notions from category theory. Restriction and induction functors.
  - (ii) Explicit construction of induction functor using equivariant sheaves.
  - (iii) Frobenius formula for the character of the induced representation.
  - (iv) Mackey's theory.
- 6.** Representations of abelian groups. Fourier transform.
- 7.** Representations of semi-direct products.
- 8.** Representations of symmetric groups.
- 9.** Representations of the Heisenberg group. Weil representation of the group  $G = SL(2, \mathbf{F}_q)$ .
- 10.** Representations of the group  $G = SL(2, \mathbf{F}_q)$ .
- 11.** Some results on finite-dimensional representations of infinite groups.
- 12.** Some results about representations of topological groups.
  - (i) Representations of commutative groups and Fourier transform.
  - (ii) Basic results about representations of the compact group  $G = SO(3)$ .