

## Algebraic Geometry and Commutative Algebra.

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Course description:

This is a year long basic course in algebraic geometry for toar sheny complemented by necessary facts from commutative algebra.

**Books.** In my exposition I mostly follow the book:

**Algebraic varieties** by G.R.Kempf, Cambridge University Press (London Math. Society, Lecture Notes Series, v.172).

Sometimes for exercises I will use the book **Introduction to commutative algebra** by M.F. Atiyah and I.G.MacDonald.

**Home assignments.** I will be giving problem assignments weekly. These problem assignments are the integral part of the course - they will contain many important points for which there is not enough time in the course itself.

The grades for home assignments will be a factor in the final grade for the course.

**Exams.** There will be a midterm exam in class and a final take home exam.

### **Syllabus of Part I.** (Spring semester 2011)

Affine algebraic varieties  
Zariski topology  
Noether's normalization lemma  
Hilbert's basis theorem and Nullstellensatz  
Projective varieties and general algebraic varieties  
Products of algebraic varieties  
Separated and complete varieties  
Decomposition into irreducible components  
Dimension - different definitions and properties  
Principal ideal theorem  
Smooth points and tangent spaces  
Degree of a projective variety  
Classical examples of algebraic varieties  
Elements of Schemes theory

### **Syllabus of Part II.** (Fall semester 2011):

Algebraic curves and their non-singular models  
Riemann-Roch theorem - elementary approach  
Sheaves  
Coherent sheaves and localization. Serre's lemma  
Cohomologies and elements of homological algebra.  
Higher cohomological operations with sheaves. Base change  
Different versions of Riemann-Roch theorem and its applications.

Jacobians of curves

Weil's proof of Riemann hypothesis for curves over finite fields.