Course description - Algebraic Theory of D-modules.

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This is an advanced course on the Algebraic Theory of D-modules. It will be quite intensive.

First I will describe basic properties of modules over the Weyl algebra $D = D_n$ (which is just the algebra of linear differential operators with polynomial coefficients in n variables).

This part will be rather elementary. I do not assume many prerequisites, though familiarity with some basic notions of algebraic geometry would be helpful (the main tool in studying D-modules is to reduce problems about D-modules to problems about modules over polynomial rings - which are in the realm of algebraic geometry).

The main new and highly non-trivial notion which emerges in this theory is the notion of a holonomic D-module. We will study this notion in details and show that it has many non-trivial applications in analysis.

In our discussion we will try to emphasize that the constructions that arise in D-module theory have very clear origin in analytic problems related to D-modules.

Later in the course I will (slowly) introduce the cohomological technique for studying D-modules. We will see that the central notion of a holonomic D-module have an equivalent description in homological terms.

Then we generalize the notion of D-module to general algebraic varieties. Of course here I assume the knowledge of basic Algebraic Geometry.

Also it turns out that the work with D-modules requires from the beginning to use more sophisticated homological technique (derived categories). We will spend some time discussing this technique.

In case of general algebraic varieties one has to work with sheaves - so I will discuss the basic properties of sheaves.

Of course at this stage I will have to assume that people know some basic properties of coherent sheaves on algebraic varieties (or at least have heard about them).

In other words, I will try to make my lectures formally self contained, but a prior knowledge of basic facts about coherent sheaves in algebraic geometry would help a lot.

I will also describe (without complete proofs) the relation of D-module theory to the standard cohomological theory of sheaves (Riemann-Hilbert correspondence).

At the end of the course I will try to describe how the language and some basic results of D-module theory allow to give a geometric interpretation and also prove some highly non-trivial results in representation theory of reductive Lie algebras and reductive Lie groups.

Books

As a reference I will use the book "Algebraic D-modules" by A.Borel et all, and probably some other more recent texts on the subject.