Theory of automorpfic functions.

Course description.

Joseph Bernstein **Fall 2012**

This is a first part of a year long course for "toar Sheny" on the theory of Automorphic forms and Automorphic Representations. This theory is now one of the focal points of Mathematics. It has very many applications in different areas of Number Theory, Physics, Combinatorics and so on.

This theory is in the process of constant development and it is not easy to formulate what are the goals of the theory. I will try to describe basic structures of the theory and main ideas developed in it. I will also try to illustrate them by examples of applications.

I will try to describe the development of this theory in quasi-historical manner, starting with the classical theory of modular forms.

In Fall semester I will mostly discuss algebraic and Representation Theoretic aspects of the theory.

In Spring semester I will try to discuss analytic aspects of the theory related to bounds for L-functions associated to automorphic representations.

I hope to be able to outline main conjectural structures of the theory evolving around Langlands' program, Generalized Riemann Hypothesis and Ramanujan conjecture. One should realize that though in many cases we can formulate rather precise conjectures describing the behavior of L-functions and automorphic representations what we can prove about these conjectures is just scratching the surface.

Books.

In my exposition I will use many sources. Here are some of them.

(1) "1-2-3 of modular forms".

(2) "Introduction to Langlands program"

Home assignments.

I will be giving problem assignments weekly. These problem assignments are the integral part of the course - they will contain

many important points for which there is not enough time in the course itself.

The grades for home assignments will be a factor in the final grade for the course.