Theory of automorphic functions

Syllabus of first part of the course (Fall semester 2012).

(1) Classical theory of modular forms for the group $SL(2, \mathbf{Z})$.
Relation with elliptic curves, Eisenstein series, Structure of the algebra of modular forms.
Fourier expansion of modular forms.
Different constructions of modular forms.
Discriminant function and cusp forms. Properties of coefficients $\tau(n)$.
Digression on Dirichlet L-functions.
Hecke operators and Hecke L-functions.
Peterson scalar product.

(2) Maass forms and related L-functions.

(3) Modular functions with respect to congruence subgroups. New forms and their L-functions. Weil's inverse theorem.

(4) Modular forms with respect to co-compact subgroups. Statement of the Jacquet-Langlands correspondence.

(5) Introduction to Hilbert modular forms.

(6) Introduction to Siegel modular forms.

(7) Automorphic forms and Representation Theory.
Digression on representations of the group $SL(2, \mathbf{R})$. Interpretation of automorphic forms in terms of automorphic representations.

(8) Adelic automorphic representations.
Adeles.
Tate's thesis and L-functions
Passing to adelic automorphic representations.
Hecke L-function of an automorphic representation.

(9) Langlands' L-function. Class field theory and Langlands program.

(10) Langlands' dual group. Langlands program and functoriality conjecture.

(11) Relation of Langlands program to Artin and Ramanujan conjectures.