SHEAR

Rotation of a Picture

Simple Rotation

The Simplest method is by using a rotation matrix

$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

Simple Rotation

$$\begin{pmatrix} \mathrm{x} \\ \mathrm{y} \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \mathrm{x - x0} \\ \mathrm{y - y0} \end{pmatrix} + \begin{pmatrix} \mathrm{x0} \\ \mathrm{y0} \end{pmatrix}$$

$$= \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \mathrm{x} \\ \mathrm{y} \end{pmatrix} - \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \mathrm{x0} \\ \mathrm{y0} \end{pmatrix} + \begin{pmatrix} \mathrm{x0} \\ \mathrm{y0} \end{pmatrix}$$

$$= \begin{pmatrix} \mathrm{x}\cos\alpha - \mathrm{y}\sin\alpha \\ \mathrm{x}\sin\alpha + \mathrm{y}\cos\alpha \end{pmatrix} - \begin{pmatrix} \mathrm{x0}\cos\alpha - \mathrm{y0}\sin\alpha - \mathrm{x0} \\ \mathrm{x0}\sin\alpha + \mathrm{y0}\cos\alpha - \mathrm{y0} \end{pmatrix}$$

Simple Rotation

We can calculate incrementally, for x+1 we get

$$\begin{pmatrix} (\mathrm{x}+1)\cos\alpha - \mathrm{y}\sin\alpha \\ (\mathrm{x}+1)\sin\alpha + y\cos\alpha \end{pmatrix} - \begin{pmatrix} \mathrm{x0}\cos\alpha - \mathrm{y0}\sin\alpha - \mathrm{x0} \\ \mathrm{x0}\sin\alpha + \mathrm{y0}\cos\alpha - \mathrm{y0} \end{pmatrix}$$

$$= \begin{pmatrix} \mathrm{x}\cos\alpha - \mathrm{y}\sin\alpha \\ x\sin\alpha + \mathrm{y}\cos\alpha \end{pmatrix} - \begin{pmatrix} \mathrm{x0}\cos\alpha - \mathrm{y0}\sin\alpha - \mathrm{x0} \\ \mathrm{x0}\sin\alpha + \mathrm{y0}\cos\alpha - \mathrm{y0} \end{pmatrix} + \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}$$

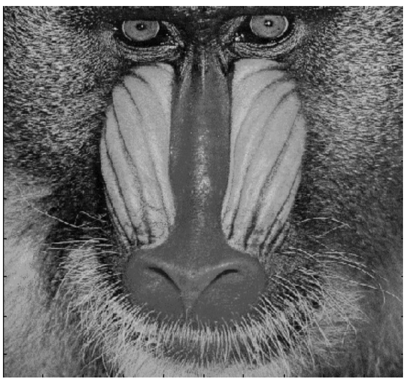
Simple Rotation

For each increment in x w $\begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}$

For each increment in y v $\begin{pmatrix} -\sin\alpha \\ \cos\alpha \end{pmatrix}$

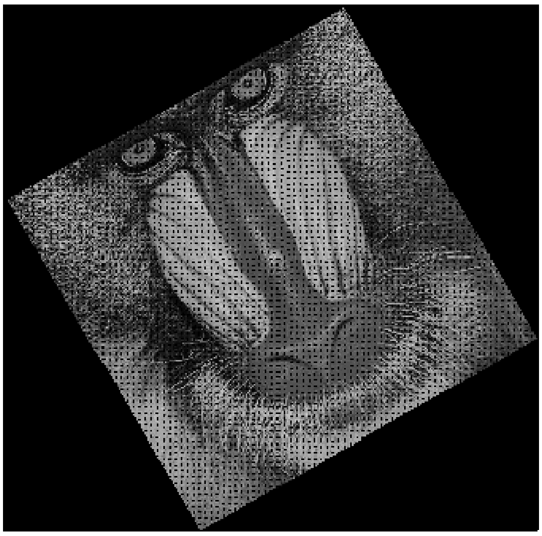
Simple Rotation

Origin



Simple Rotation

Result



Simple Rotation

Solutions

- Filter

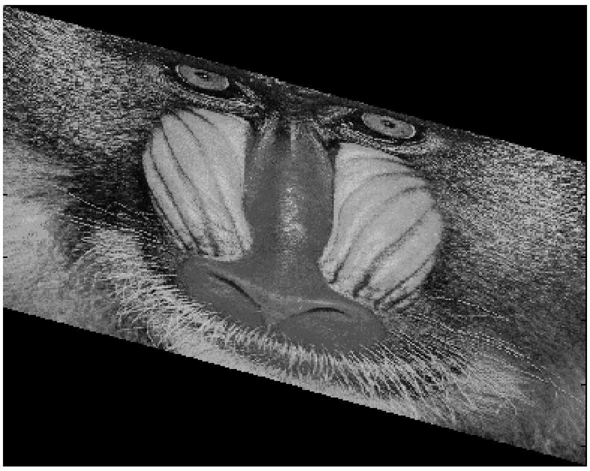
- Backward Mapping

Backword Mapping

Original Calculation $\begin{pmatrix} u \\ v \end{pmatrix} = M \begin{pmatrix} x \\ y \end{pmatrix}$

Backward mapping $\begin{pmatrix} x \\ y \end{pmatrix} = M^{-1} \begin{pmatrix} u \\ v \end{pmatrix}$

Shear



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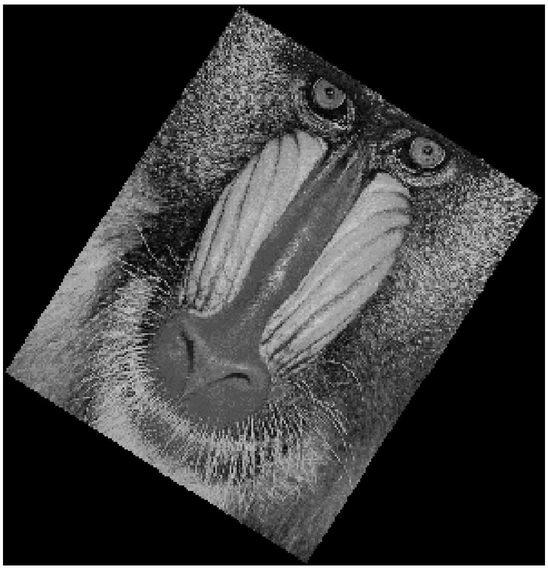
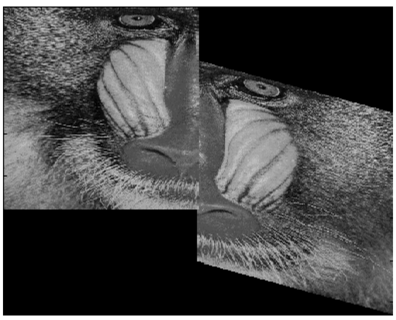


Shear

Shear-and-Scale



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Filter is still necessary, because of holes

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Advantages

- Filter - line by line, faster
- Lines calculations instead of Matrix calculation, can be implemented with special computer for better performance

Shear

Shear Transformation

$$\begin{pmatrix} u \\ v \end{pmatrix} = A \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} r \\ s \end{pmatrix} = B \begin{pmatrix} u \\ v \end{pmatrix} = B \left(A \begin{pmatrix} x \\ y \end{pmatrix} \right) = T \begin{pmatrix} x \\ y \end{pmatrix}$$

Shear

T is the Rotation Matrix

$$\begin{pmatrix} r \\ s \end{pmatrix} = T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \cos\alpha - y \sin\alpha \\ x \sin\alpha + y \cos\alpha \end{pmatrix}$$

Shear

A preserve columns

$$\begin{pmatrix} u \\ v \end{pmatrix} = A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ f(x, y) \end{pmatrix}$$

B preserve rows

$$\begin{pmatrix} r \\ s \end{pmatrix} = B\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} g(u, v) \\ v \end{pmatrix}$$

Shear

From

$$\begin{pmatrix} r \\ s \end{pmatrix} = B\begin{pmatrix} u \\ v \end{pmatrix} = B\left(A\begin{pmatrix} x \\ y \end{pmatrix}\right) = B\begin{pmatrix} x \\ f(x, y) \end{pmatrix} = \begin{pmatrix} g(x, f(x, y)) \\ f(x, y) \end{pmatrix}$$

We get $f(x, y) = s = x \sin\alpha + y \cos\alpha$

$$\begin{pmatrix} r \\ s \end{pmatrix} = T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \cos\alpha - y \sin\alpha \\ x \sin\alpha + y \cos\alpha \end{pmatrix}$$

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Finding g(u,v)

$$g(u,v) = x\cos\alpha - y\sin\alpha$$

We need to express it in terms of u,v

We know that x=u, and

$$v = f(x,y) = x\sin\alpha + y\cos\alpha$$

We get

$$y = \frac{v - x\sin\alpha}{\cos\alpha} = \frac{v - u\sin\alpha}{\cos\alpha}$$

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We put it all together and get

$$g(u,v) = u\cos\alpha - \frac{v - u\sin\alpha}{\cos\alpha}\sin\alpha = u\sec\alpha - v\tan\alpha$$

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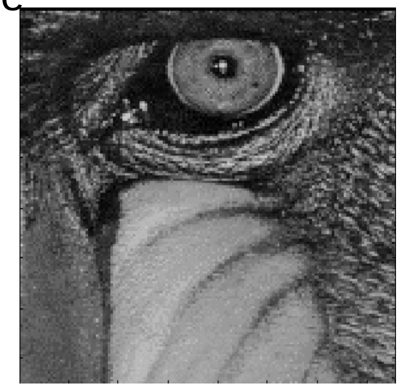
At last we get

$$A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ x\sin\alpha + y\cos\alpha \end{pmatrix}$$

$$B\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u\sec\alpha - v\tan\alpha \\ v \end{pmatrix}$$

Shear

Using a large
angle
(80 degree)



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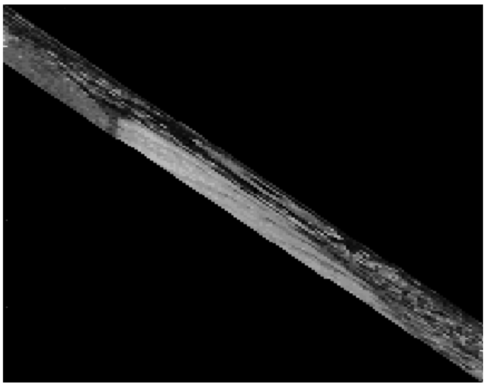
Using a large angle

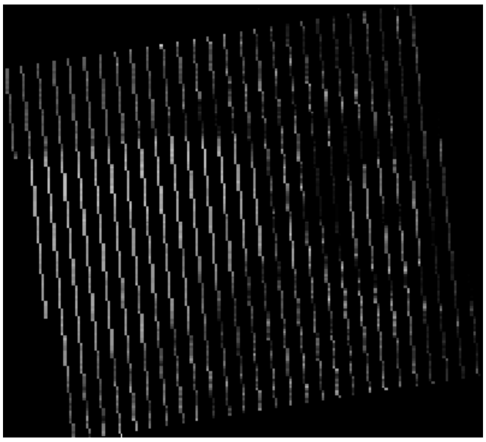
First Pass



Shear

Using a large angle

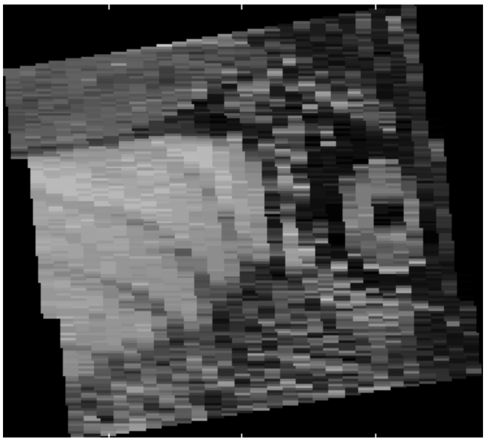
Second Pass



Shear

Using a large angle

Second Pass With Backward Mapping



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Solution:

- First, rotate in 90 degree
- Second, use shear with a small angle

Shear

We still have a scale factor in the shear which create holes

one solution is to use filter

Shear

The other solution is by using three shear transformations

$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} = \begin{pmatrix} 1 & -\tan\alpha/2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \sin\alpha & 1 \end{pmatrix}\begin{pmatrix} 1 & -\tan\alpha/2 \\ 0 & 1 \end{pmatrix}$$

We need Three passes instead of Two

Shear

Advantages

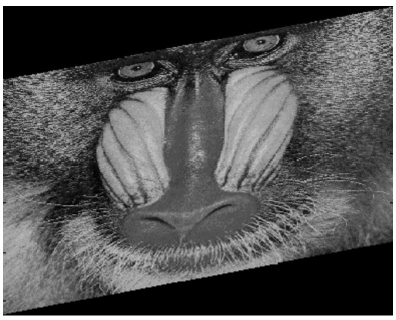
- No scale, No filter
- Much Faster, only moving lines

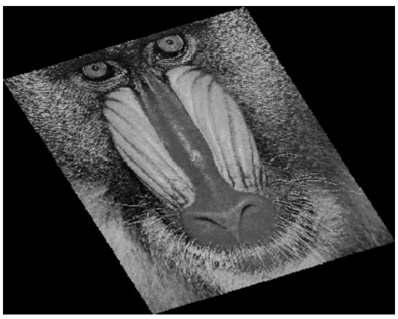
Disadvantages

- We need Three passes instead of Two

Shear

Three shears





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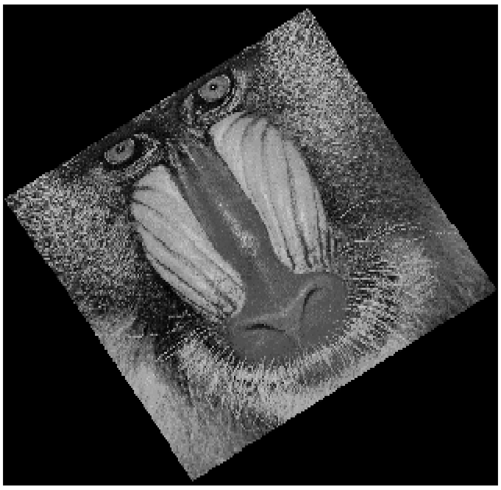

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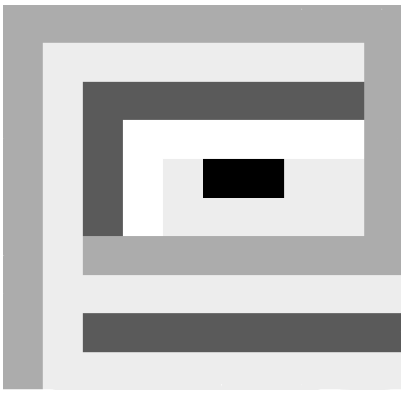

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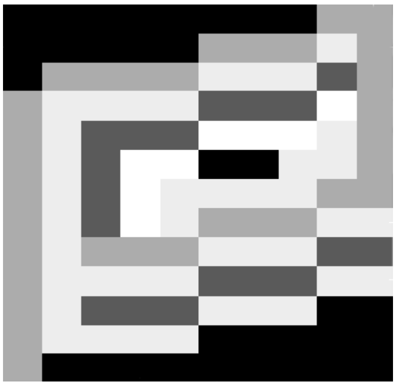

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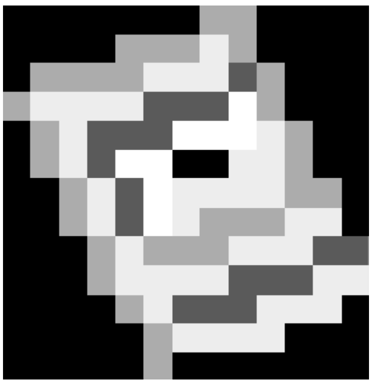

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