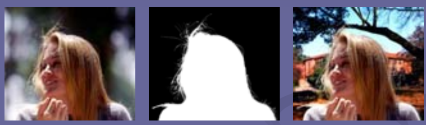
Digital Matting



Presenting:
Alon Gamliel, Tel-Aviv University, May 2006

Outline

1. Introduction to Digital Matting
2. Bayesian Matting
3. Poisson Matting
4. A Closed Form Solution to Matting

Introduction to Digital Matting

Goal: Separate a foreground object from the background

Original Photo → Object with new background

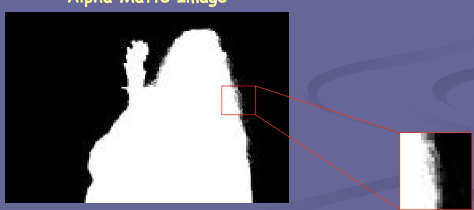
Introduction to Digital Matting

Problem: Intricate boundaries – hair strands and fur.

Introduction to Digital Matting

Solution: Opacity

Alpha Matte Image



Compositing equation:

$$C = \alpha * F + (1-\alpha) * B$$

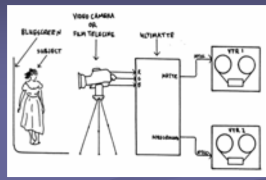
Compositing equation:

$$C = \alpha F + (1-\alpha)B \quad \begin{cases} CR = \alpha * FR + (1-\alpha) * BR \\ CG = \alpha * FG + (1-\alpha) * BG \\ CB = \alpha * FB + (1-\alpha) * BB \end{cases}$$

Solve:
Given C, find (F,B,α) for each pixel

The Problem:
3 equations, 7 unknowns

Blue Screen Matting

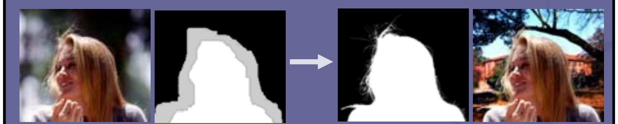


Chroma Keying, "Blue Screen" Matting

- Known Background
- Controlled Scene

Statistical Methods

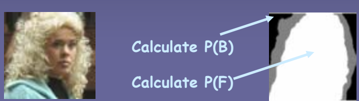
Berman et al. 2000, Ruzon and Tomasi 2000, Chuang et al. 2001



Input | User Supplied Trimap | Alpha Matte Image | Compositing

Statistical Methods

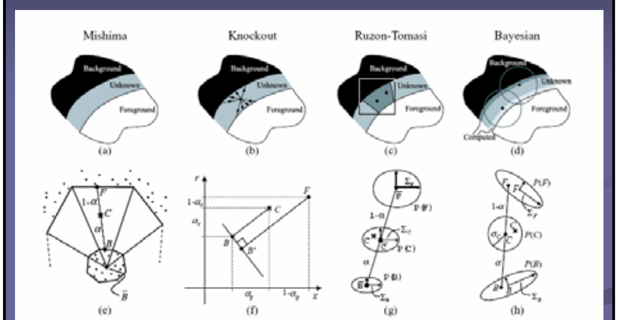
1. gather statistical information from user trimap



2. solve for F, B, α in the unknown region

Rely on good separation in color space

Color Model Acquisition - Summary



Blue Screen Geometric Model | Natural Matting Geometric Model | Natural Matting Probabilistic Model | Natural Matting Bayesian Model

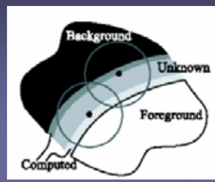
Bayesian Matting

"Video Matting of Complex Scenes", SIGGRAPH 2002
"A Bayesian Approach to Digital Matting", CVPR 2001

Chuang, Agarwala, Curless, Salesin, Szeliski

Bayesian Matting Overview

- 1. Start with a user trimap



- 2. Solve for boundaries of the unknown region
  - Estimate F,B,α using probabilistic framework, relying on nearest pixels from trimap
- 3. Refine trimap
- 4. Back to (2)

Bayesian Matting

$$(F,B,\alpha) = \arg\max_{F,B,\alpha} P(F,B,\alpha \mid C)$$

- Find most likely (F, B, α) values for each pixel.

$$= \arg\max_{F,B,\alpha} P(C \mid F,B,\alpha)P(F)P(B)P(\alpha)/P(C)$$

- Apply Bayes' Rule

$$= \arg\max_{F,B,\alpha} P(C \mid F,B,\alpha)P(F)P(B)P(\alpha)$$

- P(C) is constant with F,B, α

$$= \arg\max_{F,B,\alpha} L(C \mid F,B,\alpha) + L(F) + L(B) + L(\alpha)$$

- Use log-likelihood

Bayesian Matting – Details

- Estimating L(C|F,B,α)

$$L(C \mid F,B,\alpha) \quad = \quad -\|C - \alpha F - (1-\alpha)B\|^2/\sigma_C^2$$

  - This log-likelihood models error in the measurement of C and corresponds to a gaussian probability distribution centered at $\mu=\alpha F+(1-\alpha)B$ with standard deviation σ_C

Estimating Color Correspondence

- The probability that color c agree with gaussian color model, parameterized with μ, Σ

$$f(c|\mu,\Sigma) = \frac{1}{\sqrt{(2\pi)^d \det(\Sigma)}} \exp\left(-\frac{(c-\mu)^T \Sigma^{-1} (c-\mu)}{2}\right)$$

- Taking the maximum logarithm (in respect to c)

$$L(c|\mu,\Sigma) = -(c-\mu)^T \Sigma^{-1} (c-\mu)$$

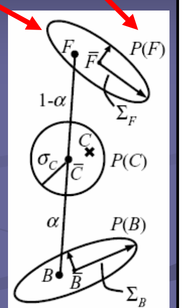
Bayesian Matting – Details

Nearest known F points (weighted)

L(F)

- Estimating L(F),L(B)
  - How well estimated F,B correspond to F/B color model

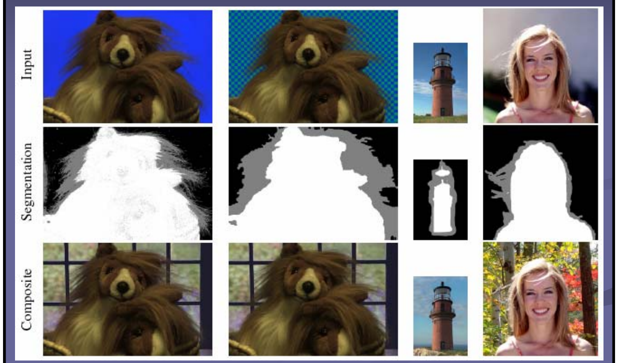
$$L(F) \quad = \quad -(F-\overline{F})^T \, \Sigma_F^{-1} \, (F-\overline{F}) \, / \, 2$$



Bayesian Matting – Details

- Eliminating L(α)
  - With no other good assumption, assume L(α) is constant for every possible α
- Solving:
  - Derive from **argmax(L(C|F,B,α)+L(F)+L(B))** two sets of linear equations
  - Solve iteratively, alternating the sets, assuming that α **or** F/B are constant each iteration

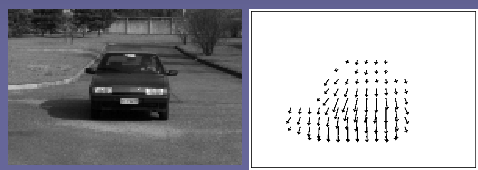
Bayesian Matting – Results



Video Matting

- Problem
  - Bayesian matting requires a trimap for each frame
  - Drawing trimaps is labor-intensive
- Solution
  - Draw trimaps only for key-frames
  - Interpolate trimaps between key-frames
  - How to interpolate? → Optical Flow

Optical Flow



$$C_{i+1}(\mathbf{x}) = C_i(\mathbf{x}+\mathbf{u})$$

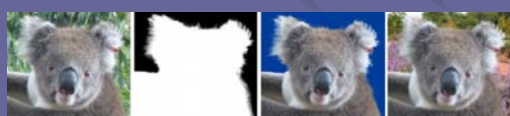
- C(x) : color at pixel coord. x
- u(x) : velocity of the pixel at x – called *flow field*

Video Result

Poisson Matting

"Poisson Matting", SIGGRAPH 2004

Sun, Jia, Tang, Shum

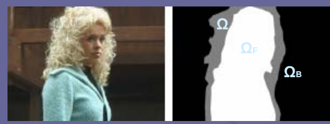


Poisson Matting Overview

- Assumption: The matting gradients mimic the image gradients
- 1. Start with a user trimap
- 2. Estimate alpha values in unknown area, based on image gradients and known alpha values
- 3. Refine trimap
- 4. Back to (2)

Initializing Iterative Process

- Start from a user-defined trimap



- For each p in Ω, measure F&B from the nearest pixels in Ω_F & Ω_B

Estimating Matting Field

$$I = \alpha F + (1-\alpha)B$$

$$\nabla I = (F-B)\nabla\alpha + \alpha\nabla F + (1-\alpha)\nabla B$$

- Assume smooth foreground & background

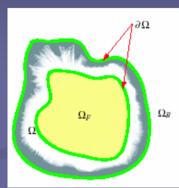
$$\alpha\nabla F + (1-\alpha)\nabla B << (F-B)\nabla\alpha$$

- We get:

$$\nabla\alpha \approx \frac{1}{F-B}\nabla I$$

Calculating α

- Ω – unknown area
- Ω_F – trimap foreground area
- Ω_B– trimap background area



- Solve:

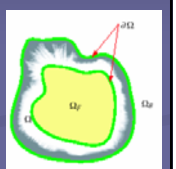
$$\alpha^* = \arg\min_{\alpha} \int\int_{p\in\Omega} ||\nabla\alpha_p - \frac{1}{F_p - B_p}\nabla I_p||^2 dp$$

- F_p/B_p are taken from nearest pixels at Ω_F / Ω_B

Calculating α

- Minimizing is equivalent to solving the Poisson equation:

$$\Delta\alpha = \frac{\partial^2\alpha}{\partial x^2} + \frac{\partial^2\alpha}{\partial y^2} = div\left(\frac{\nabla I}{F-B}\right)$$



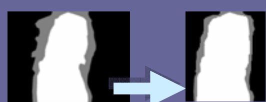
- Assuming Dirichlet boundary condition

$$\hat{\alpha}_p|_{\partial\Omega} = \begin{cases} 1 & p\in\Omega_F \\ 0 & p\in\Omega_B \end{cases}$$

- F, B and ∇I are measured in grayscale
- Solve linear system

Refinement

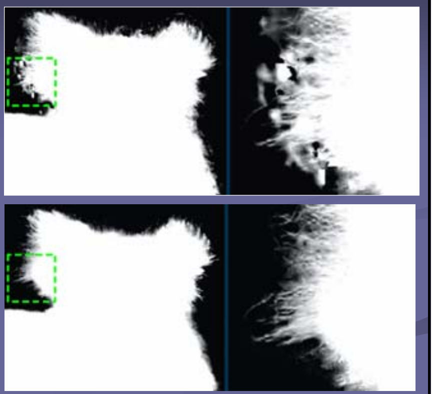
- Ω_{F+} = { p in Ω, s.t. $\alpha_p > 0.95$ ($I_p \sim F_p$) }
- Ω_{B+} = { p in Ω, s.t. $\alpha_p < 0.05$ ($I_p \sim B_p$) }
- Generate a new trimap



- Iterate for "few" iterations

Results

- Bayesian
- Poisson
- Given Image

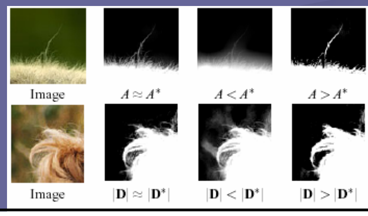


Error Effects

$$\nabla I = (F-B)\nabla\alpha + \alpha\nabla F + (1-\alpha)\nabla B$$

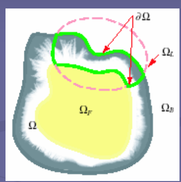
$$\cancel{\alpha\nabla F + (1-\alpha)\nabla B << (F-B)\nabla\alpha}$$

$$\nabla\alpha = \frac{1}{F-B}\left(\nabla I - \alpha\nabla F - (1-\alpha)\nabla B\right) = A(\nabla I - D)$$



Calculating α in the local case

- Ω – unknown area
- Ω_F – trimap foreground area
- Ω_B – trimap background area
- Ω_L – user selected area



- Solve:

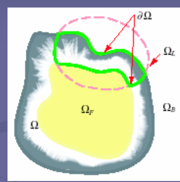
$$\alpha^* = \arg\min_{\alpha} \iint_{p\in\Omega_L\cap\Omega} \|\nabla\alpha_p - A_p(\nabla I_p - \mathbf{D}_p)\|^2 dp.$$

$$A = \frac{1}{F-B} \qquad D = \alpha\nabla F + (1-\alpha)\nabla B$$

Calculating α in the local case

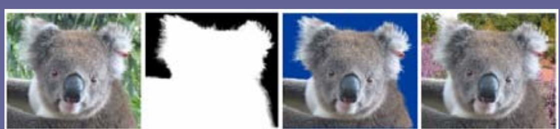
- Assuming Dirichlet boundary condition

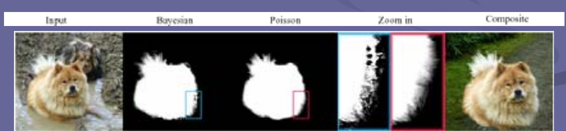
$$\hat{\alpha}_p|_{\partial\Omega} = \begin{cases} 1 & p\in\Omega_F \\ 0 & p\in\Omega_B \\ \alpha_g & p\in\Omega \end{cases}$$



- α_g – was calculated at the global case
- No local discontinuity can be seen

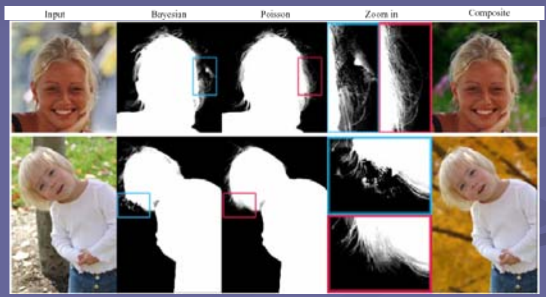
Results





Results

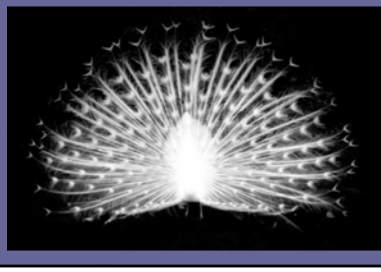
Results



A Closed Form Solution to Matting

"A Closed Form Solution to Natural Image Matting", CVPR 2006

Levin, Lischinski, Weiss



Recall Poisson Matting…

$$I = \alpha F + (1-\alpha)B$$

$$\nabla I = (F-B)\nabla\alpha + \alpha\nabla F + (1-\alpha)\nabla B$$

- Assume smooth foreground & background

$$\alpha\nabla F + (1-\alpha)\nabla B << (F-B)\nabla\alpha$$

- We get:

$$\nabla\alpha \approx \frac{1}{F-B}\nabla I$$

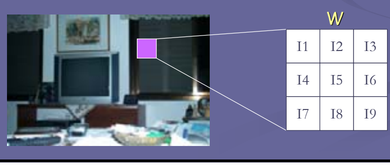
Estimating Matting Field

$$I = \alpha F + (1-\alpha)B$$

- Assume smooth foreground & background

For every window W

$\alpha_i \approx aI_i + b$ for each i in W



$a = 1/(F-B)$

$b = -B/(F-B)$

Estimating Matting Field

- Minimize J:

$$J(\alpha, a, b) = \sum_{j \in I}\left(\sum_{i \in w_j}(\alpha_i - a_j I_i - b_j)^2 + \varepsilon a_j^2\right)$$

$$J(\alpha) = \min_{a,b} J(\alpha, a, b)$$

- ε is a regulation term

Closed Form Solution

- It can be proven that:

$$J(\alpha) = \alpha^T L\, \alpha$$

- Where L is NxN matrix, and computed directly:

$$\sum_{k|(i,j)\in w_k}\left(\delta_{ij} - \frac{1}{|w_k|}\left(1 + \frac{1}{\frac{\varepsilon}{|w_k|} + \sigma_k^2}(I_i - \mu_k)(I_j - \mu_k)\right)\right)$$

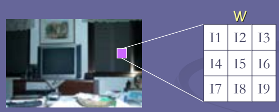
- δ – delta function
- μ, σ – window parameters

Moving to Color Images

For every window W

$\alpha_i \approx a^R I^R_i + a^G I^G_i + a^B I^B_i + b$

for each i in W



- Derive a^R, a^G, a^B, b from : $F = \beta F_1 + (1-\beta)F_2$, $B = \gamma B_1 + (1-\gamma)B_2$
- This relaxes the assumption that foreground & background are smooth

Moving to Color Images

- L is still NxN matrix, but computed differently:

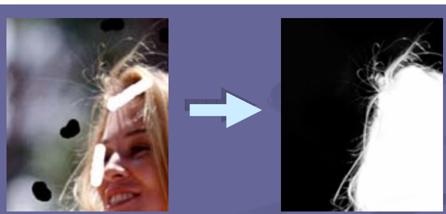
$$\sum_{k|(i,j)\in w_k}\left(\delta_{ij}-\frac{1}{|w_k|}\left(1+(I_i-\mu_k)(\Sigma_k+\frac{\varepsilon}{|w_k|}I_3)^{-1}(I_j-\mu_k)\right)\right)$$

- δ – delta function
- μ, Σ– window parameters

Solving for α, F, B

- Solving for α, based on user constraints (scribbles)

$$\alpha = \operatorname{argmin}\ \alpha^T L\,\alpha, \quad \text{s.t. } \alpha_i = s_i,\ \forall i \in S$$



- Can be transformed to $A\alpha = B$ linear problem

Solving for α, F, B

- To find F,B, solve a set of linear equations
  - Correspondence term (for each pixel)

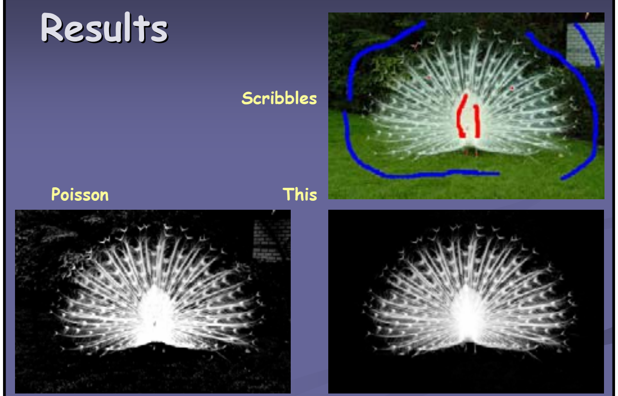
$$(\alpha F + (1-\alpha)B - I)^2$$

  - Smoothness terms (for each pixel)

$$\alpha\|F(x,y)-F(x+1,y)\|^2 +$$
$$\alpha\|F(x,y)-F(x,y+1)\|^2 +$$
$$(1-\alpha)\|B(x,y)-B(x+1,y)\|^2 +$$
$$(1-\alpha)\|B(x,y)-B(x+1,y)\|^2$$

Results

Results



More details…

- We wish to solve: $\begin{cases} \alpha^* = \arg\min(\alpha^T L \alpha) \\ \alpha^i = s^i & \forall i \in S \end{cases}$
- Which is like:

$$\arg\min\left(\alpha^T L\alpha + \lambda\sum_{i\in S}(\alpha^i - s^i)^2\right) \qquad \lambda \to \infty$$

- Differentiate: $\left(\bar{\bar{L}} + \lambda\bar{\bar{D}}\right)\bar{\alpha} = \lambda\bar{B}$
  - D is a diagonal matrix, where d_{ii} == 1 iff i in S
  - B is a vector, where B_i==known alpha value iff i in S
- Cholesky Decomposition:

$$RR^T\bar{\alpha} = \lambda\bar{B}$$

Future Work (my thesis)

- Interactive Matting
  - Incorporate additional user scribbles directly into Aα = B linear problem
- Integrating global color model
  - Statistical analysis of user scribbles
  - Based on this analysis add more alpha constraints
- Solving for video
  - Goal: Keep a minimal user interface framework
  - Suggestion: Feature-based expansion of user scribbles
  - Goal: Retrieving coherent results between frames
  - Suggestion: Globally solve for frame pairs
- Live presentation…

Summary

	Bayesian	Poisson	Closed Form
Hints	Local color analysis	Image gradients	Image windows
Assumptions	Color separation, camera quality	Locally smooth foreground and background	Locally linear foreground and background
User Data	Trimap	Trimap	Scribbles
Solving Method	Iterative	Iterative	Direct
Computation Time (640x480)	~30 secs	~5 secs	~60 secs