Poisson Image Editing

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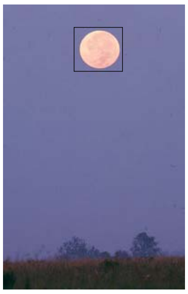
Goals

- Seamlessly importing (cloning) transparent and opaque source image regions into a destination image.
- Seamless modification of appearance of the image within a selected region.

Input Images



source image

target image

Simple Cloning Result

Poisson Seamless Cloning Result

Some More Results

source images

target image

Some More Results

source images

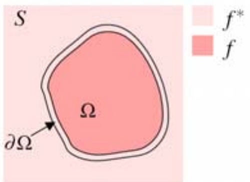
simple cloning

Some More Results

source images

Poisson seamless cloning

Interpolation



- S: a closed subset of R^2
- Ω: a closed subset of S, with boundary $\partial\Omega$
- f*: known scalar function over $S \setminus \Omega$
- f: unknown scalar function over Ω

Membrane Interpolant

- Solve the following minimization problem:

$$\min_{f} \iint_{\Omega} \left\| \nabla f \right\|^2$$

- subject to Dirichlet boundary conditions:

$$f\big|_{\partial\Omega} = f^*\big|_{\partial\Omega}$$

Euler-Lagrange Equation

- A fundamental equation of *calculus of variations*, which states that if J is defined by an integral of the form: $J = \int F(x, f, f_x)\, dx$
- then J has a stationary value if the following differential equation is satisfied:

$$\frac{\partial F}{\partial f} - \frac{d}{dx}\frac{\partial F}{\partial f_x} = 0$$

Euler-Lagrange (continued)

- In our case: $F = \left\| \nabla f \right\|^2 = \left(f_x^{\ 2} + f_y^{\ 2} \right)$
- The equation is: $\frac{\partial F}{\partial f} - \frac{d}{dx}\frac{\partial F}{\partial f_x} - \frac{d}{dy}\frac{\partial F}{\partial f_y} = 0$
- $\frac{\partial F}{\partial f} = 0$ and $\frac{d}{dx}\frac{\partial F}{\partial f_x} = \frac{d}{dx} 2 f_x = 2\frac{\partial^2 f}{\partial x^2}$
- So we get: $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \Delta f = 0$

Smooth Image Completion

Euler-Lagrange:

$$\arg\min_{f} \iint_{\Omega} |\nabla f|^2 \quad s.t. \; f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

The minimum is achieved when:

$$\Delta f = 0 \; over \; \Omega \quad s.t. \; f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

Discrete Approximation

$$\Delta f = 0 \; over \; \Omega \quad s.t. \; f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial f}{\partial x} \cong f_{x+1,y} - f_{x,y} \qquad \frac{\partial^2 f}{\partial x^2} \cong f_{x+1,y} - 2f_{x,y} + f_{x-1,y}$$

$$\Delta f(x,y) \cong f_{x+1,y} - 2f_{x,y} + f_{x-1,y} + f_{x,y+1} - 2f_{x,y} + f_{x,y-1} =$$
$$= f_{x+1,y} + f_{x-1,y} + f_{x,y+1} + + f_{x,y-1} - 4f_{x,y} = 0$$

Solving

- Each $f_{x,y}$ is an unknown variable x_i, total of N variables (covering the unknown pixels)

$f_{x,y-1}+f_{x-1,y}-4f_{x,y}+f_{x+1,y}+f_{x,y+1}=0$ or $x_{i-w}+x_{i-1}-4x_i+x_{i+1}+x_{i+w}=0$

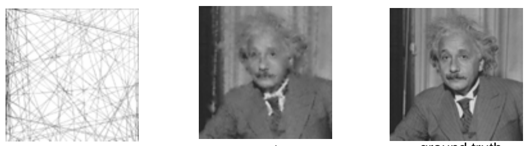
- Reduces to the sparse algebraic system:

$$\underbrace{\begin{pmatrix} 1 & & 1 & -4 & 1 & & 1 & & \\ & 1 & & 1 & -4 & 1 & & 1 & \\ & & 1 & & 1 & -4 & 1 & & 1 \\ & & & & & & & & \end{pmatrix}}_{N \times N} \begin{pmatrix} x_1 \\ x_2 \\ \cdots \\ \\ x_N \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ b_1 \\ b_2 \\ 0 \\ 0 \\ 0 \\ b_3 \\ 0 \end{pmatrix}$$

Known values of f() contribute to the left side

$x_{i-w}+x_{i-1}-4x_i+x_{i+1}=-f(x,y+1)$

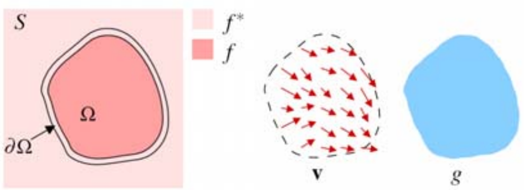
Wow result



input result ground truth

Guided Interpolation

- Laplace equation yields a smooth interpolant.
- Idea: use a guiding vector field **v**.



Poisson Cloning: "Guiding" the completion

- We can guide the completion to fill the hole using gradients from another source image
- Reverse: Seek a function f whose gradients are closest to the gradients of the source image

Guided Interpolant

- Solve the following minimization problem:

$$\min_f \iint_\Omega \|\nabla f - G\|^2$$

- subject to Dirichlet boundary conditions:

$$f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

Poisson Equation

- This time the Euler-Lagrange equation reduces to the Poisson equation:

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{\partial G_x}{\partial x} + \frac{\partial G_y}{\partial y}$$

- written more concisely as:

$$\Delta f = \mathrm{div}\ G$$

Poisson Cloning

Denote $G = \nabla source\ \ image$

$$\arg\min_f \iint_\Omega |\nabla f - G|^2 \ \ s.t.\ f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

$$\Delta f = div\, G \ over\, \Omega \ \ s.t.\ f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

(backward difference)

$$div\, G = \frac{\partial G}{dx} + \frac{\partial G}{dy} \cong G_x(x,y) - G_x(x-1,y) + G_y(x,y) - G_y(x,y-1)$$

Poisson Cloning (Solving)

Each $f_{x,y}$ is a variable x_i as before, solving

$f_{x,y-1}+f_{x-1,y}-4f_{x,y}+f_{x+1,y}+f_{x,y+1}=divG(x,y)$

Or

$x_{i-w}+x_{i-1}-4x_i+x_{i+1}+x_{i+w}=divG(x,y)$

As before it reduces to a sparse algebraic system

Seamless Cloning

- Choose the gradient field of the source image as the guiding vector field **G**.

Concealment

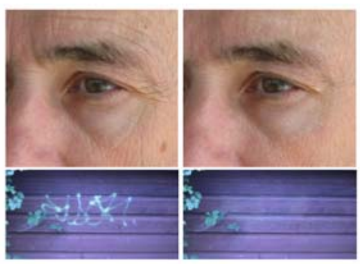
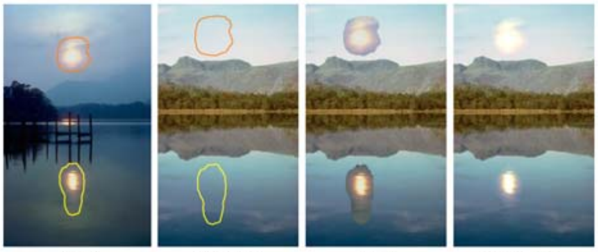


Figure 2: **Concealment**. By importing seamlessly a piece of the background, complete objects, parts of objects, and undesirable artifacts can easily be hidden. In both examples, multiple strokes (not shown) were used.

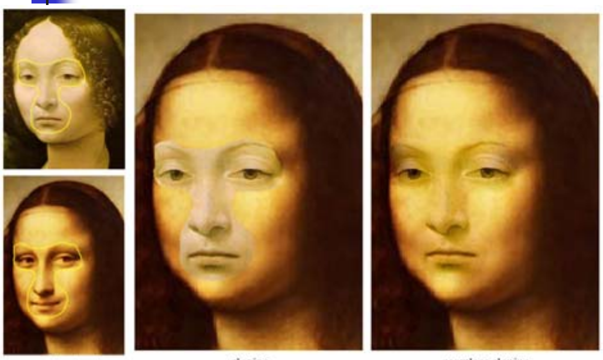


sources — destinations — cloning — seamless cloning

Insertion

sources/destinations — cloning — seamless cloning

Feature Exchange



source/destination — cloning — seamless cloning

Texture Swapping

swapped textures

Mixing Gradients

- Sometimes it is desirable to define the guiding field as a mixture of source and target gradient fields.
  - Option A: linear combination of the gradient fields
  - Option B: strongest gradient wins:

$$\mathbf{v}(\mathbf{x}) = \begin{cases} \nabla f^*(\mathbf{x}) & \text{if } |\nabla f^*(\mathbf{x})| > |\nabla g(\mathbf{x})| \\ \nabla g(\mathbf{x}) & \text{otherwise} \end{cases}$$

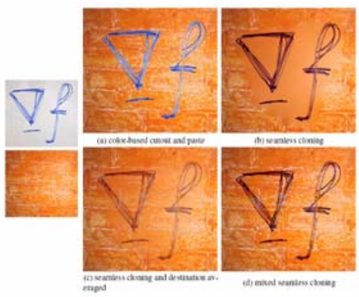


Figure 6: **Inserting objects with holes.** (a) The classic method, color-based selection and alpha masking might be time consuming and often leaves an undesirable halo; (b-c) seamless cloning, even averaged with the original image, is not effective; (d) mixed seamless cloning based on a loose selection proves effective.

More Mixtures

Figure 7: **Inserting transparent objects**. Mixed seamless cloning facilitates the transfer of partly transparent objects, such as the rainbow in this example. The non-linear mixing of gradient fields picks out whichever of source or destination structure is the more salient at each location.

More Mixtures

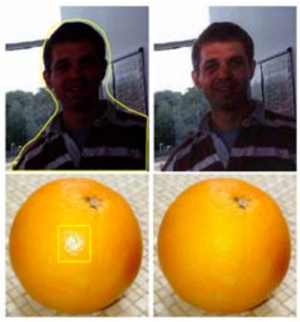
Figure 8: **Inserting one object close to another**. With seamless cloning, an object in the destination image touching the selected region Ω bleeds into it. Bleeding is inhibited by using mixed gradients as the guidance field.

Selection Editing

- Texture flattening: use target gradient field only where edges are detected:

$$\mathbf{v}(\mathbf{x}) = M(\mathbf{x})\nabla f^*(\mathbf{x})$$

Local Illumination Changes



Local Color Changes

Figure 11: **Local color changes**. Left: original image showing selection Ω surrounding loosely an object of interest; center: background decolorization done by setting g to the original color image and f^* to the luminance of g; right: recoloring the object of interest by multiplying the RGB channels of the original image by 1.5, 0.5, and 0.5 respectively to form the source image.

Seamless Tiling

Figure 12: **Seamless tiling**. Setting periodic boundary values on the border of a rectangular region before integrating with the Poisson solver yields a tileable image.