3D Polygon Rendering Pipeline

Thomas Funkhouser
Princeton University
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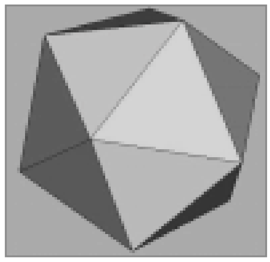
3D Polygon Rendering

- Many applications use rendering of 3D polygons with direct illumination



3D Polygon Rendering

- Many applications use rendering of 3D polygons with direct illumination

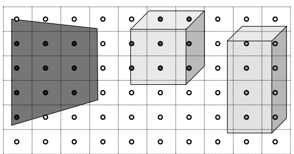
Quake II
(Id Software)

3D Polygon Rendering

- Many applications use rendering of 3D polygons with direct illumination

Ray Casting Revisited

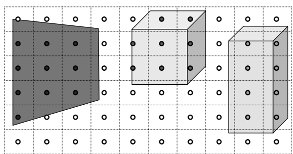
- For each sample …
  - Construct ray from eye position through view plane
  - Find first surface intersected by ray through pixel
  - Compute color of sample based on surface radiance



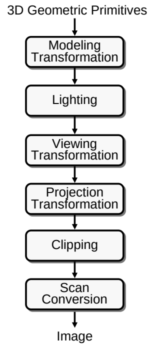
More efficient algorithms utilize spatial coherence!

3D Polygon Rendering

- What steps are necessary to utilize spatial coherence while drawing these polygons into a 2D image?

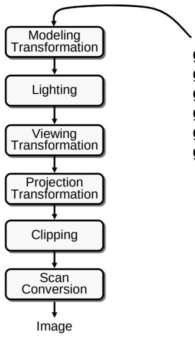


3D Rendering Pipeline (for direct illumination)



This is a pipelined sequence of operations to draw a 3D primitive into a 2D image

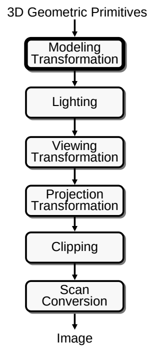
Example: OpenGL




```
glBegin(GL_POLYGON);
glVertex3f(0.0, 0.0, 0.0);
glVertex3f(0.0, 0.0, 0.0);
glVertex3f(0.0, 0.0, 0.0);
glVertex3f(0.0, 0.0, 0.0);
glEnd();
```

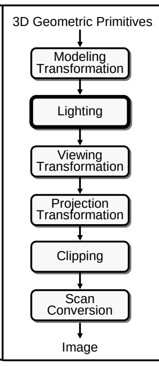
OpenGL executes steps of 3D rendering pipeline for each polygon

3D Rendering Pipeline (for direct illumination)



Transform into 3D world coordinate system

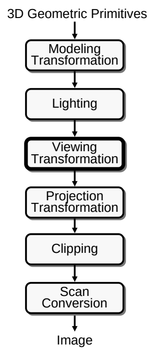
3D Rendering Pipeline (for direct illumination)



Transform into 3D world coordinate system

Illuminate according to lighting and reflectance

3D Rendering Pipeline (for direct illumination)

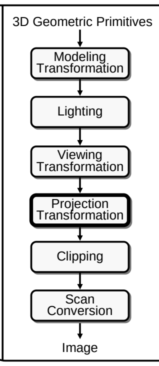


Transform into 3D world coordinate system

Illuminate according to lighting and reflectance

Transform into 3D camera coordinate system

3D Rendering Pipeline (for direct illumination)



Transform into 3D world coordinate system

Illuminate according to lighting and reflectance

Transform into 3D camera coordinate system

Transform into 2D camera coordinate system

3D Rendering Pipeline (for direct illumination)

3D Geometric Primitives

Modeling Transformation	Transform into 3D world coordinate system
Lighting	Illuminate according to lighting and reflectance
Viewing Transformation	Transform into 3D camera coordinate system
Projection Transformation	Transform into 2D camera coordinate system
Clipping	Clip primitives outside camera's view
Scan Conversion	

Image

3D Rendering Pipeline (for direct illumination)

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Image

Transformations

3D Geometric Primitives

Modeling Transformation	**Transform** into 3D world coordinate system
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Image

Transformations

p(x,y,z)

3D Object Coordinates

Modeling Transformation

3D World Coordinates

Viewing Transformation

3D Camera Coordinates

Projection Transformation

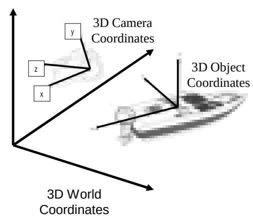
2D Screen Coordinates

Window-to-Viewport Transformation

2D Image Coordinates

p'(x',y')

Transformations map points from one coordinate system to another



Viewing Transformations

p(x,y,z)

3D Object Coordinates

Modeling Transformation

3D World Coordinates

Viewing Transformation

3D Camera Coordinates

Projection Transformation

2D Screen Coordinates

Window-to-Viewport Transformation

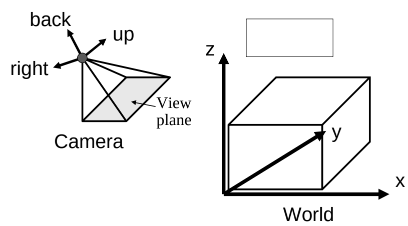
2D Image Coordinates

p'(x',y')

} Viewing Transformations

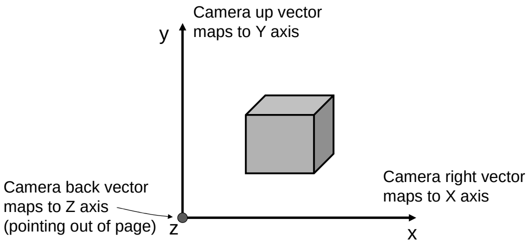
Viewing Transformation

- Mapping from world to camera coordinates
  - Origin moves to eye position
  - Up vector maps to Y axis
  - Right vector maps to X axis



Camera Coordinates

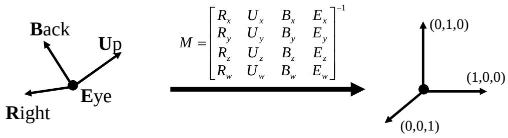
- Canonical coordinate system
  - Convention is right-handed (looking down -z axis)
  - Convenient for projection, clipping, etc.



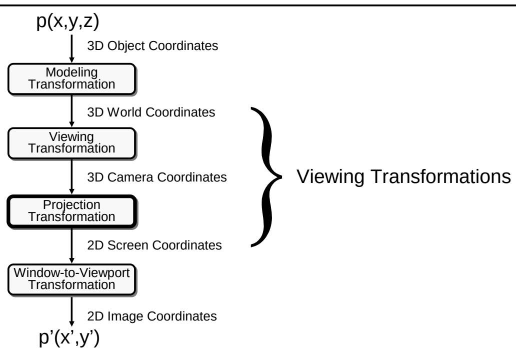
Viewing Transformation

- Transformation matrix maps camera basis vectors to canonical vectors in camera coordinate system

$$M = \begin{bmatrix} R_x & U_x & B_x & E_x \\ R_y & U_y & B_y & E_y \\ R_z & U_z & B_z & E_z \\ R_w & U_w & B_w & E_w \end{bmatrix}^{-1}$$

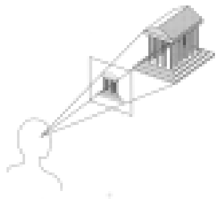


Viewing Transformations

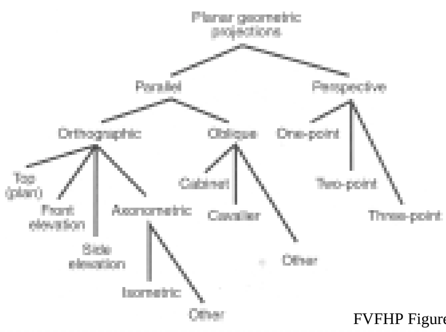


Projection

- General definition:
  - Transform points in *n*-space to *m*-space (*m*<*n*)
- In computer graphics:
  - Map 3D camera coordinates to 2D screen coordinates

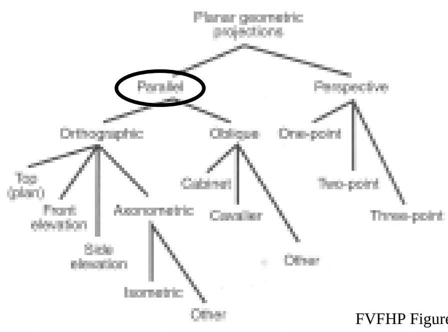


Taxonomy of Projections



FVFHP Figure 6.10

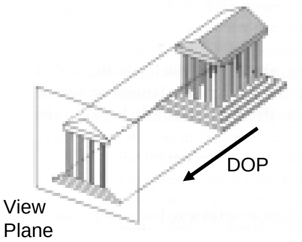
Taxonomy of Projections



FVFHP Figure 6.10

Parallel Projection

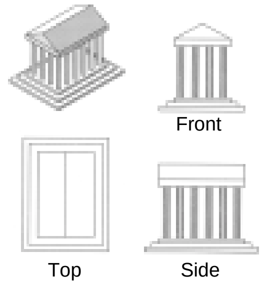
- Center of projection is at infinity
  - Direction of projection (DOP) same for all points



Angel Figure 5.4

Orthographic Projections

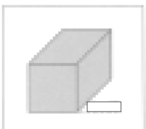
- DOP perpendicular to view plane



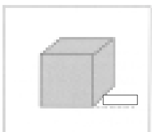
Angel Figure 5.5

Oblique Projections

- DOP not perpendicular to view plane



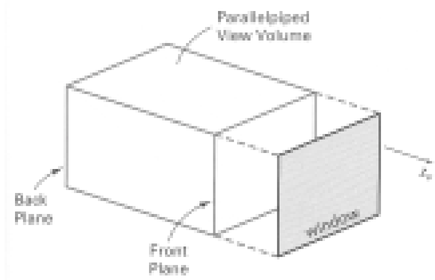
Cavalier
(DOP at 45^{o})



Cabinet
(DOP at 63.4^{o})

H&B Figure 12.24

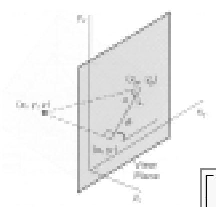
Parallel Projection View Volume



H&B Figure 12.30

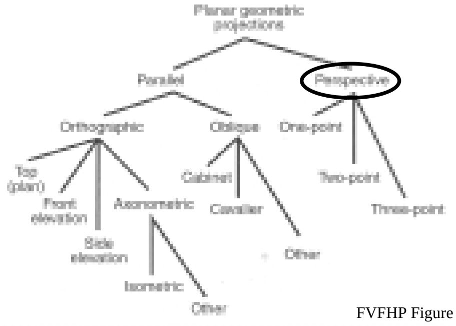
Parallel Projection Matrix

- General parallel projection transformation:



$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} 1 & 0 & L_1\cos\phi & 0 \\ 0 & 1 & L_1\sin\phi & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

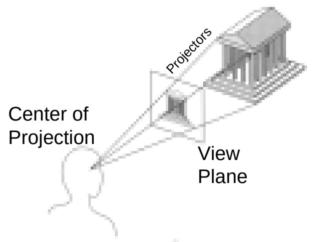
Taxonomy of Projections



FVFHP Figure 6.10

Perspective Projection

- Map points onto “view plane” along “projectors” emanating from “center of projection” (COP)



Angel Figure 5.9

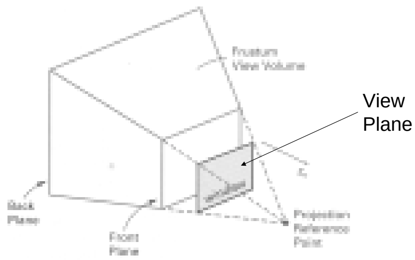
Perspective Projection

- How many vanishing points?

3-Point Perspective | 2-Point Perspective | 1-Point Perspective

Angel Figure 5.10

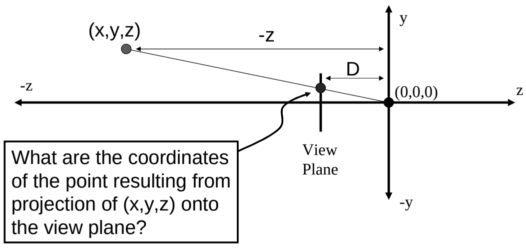
Perspective Projection View Volume



H&B Figure 12.30

Perspective Projection

- Compute 2D coordinates from 3D coordinates with similar triangles



Perspective Projection

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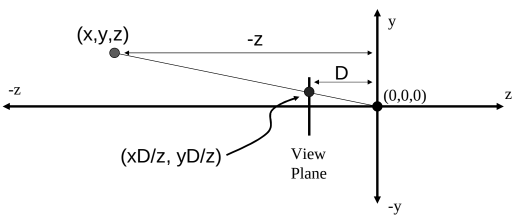


Perspective Projection Matrix

- 4x4 matrix representation?

$$x_s = x_c D / z_c$$
$$y_s = y_c D / z_c$$
$$z_s = D$$
$$w_s = 1$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Perspective Projection Matrix

- 4x4 matrix representation?

$$x_s = x_c D / z_c \qquad x' = x_c$$
$$y_s = y_c D / z_c \qquad y' = y_c$$
$$z_s = D \qquad z' = z_c$$
$$w_s = 1 \qquad w' = z_c / D$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

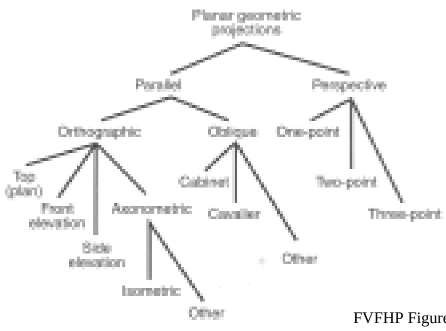
Perspective Projection Matrix

- 4x4 matrix representation?

$$x_s = x_c D / z_c \qquad x' = x_c$$
$$y_s = y_c D / z_c \qquad y' = y_c$$
$$z_s = D \qquad z' = z_c$$
$$w_s = 1 \qquad w' = z_c / D$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/D & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

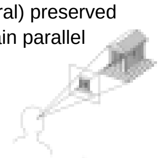
Taxonomy of Projections



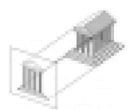
FVFHP Figure 6.10

Perspective vs. Parallel

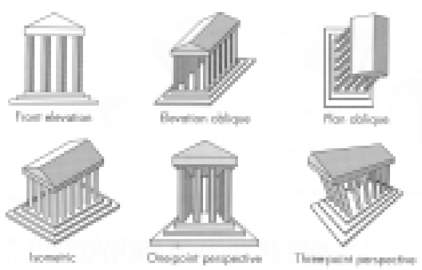
- Perspective projection
  - \+ Size varies inversely with distance - looks realistic
  - – Distance and angles are not (in general) preserved
  - – Parallel lines do not (in general) remain parallel



- Parallel projection
  - \+ Good for exact measurements
  - \+ Parallel lines remain parallel
  - – Angles are not (in general) preserved
  - – Less realistic looking



Classical Projections



Angel Figure 5.3

Summary

- Camera transformation
  - Map 3D world coordinates to 3D camera coordinates
  - Matrix has camera vectors as rows
- Projection transformation
  - Map 3D camera coordinates to 2D screen coordinates
  - Two types of projections:
    - Parallel
    - Perspective

Next Time

3D Geometric Primitives

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Image