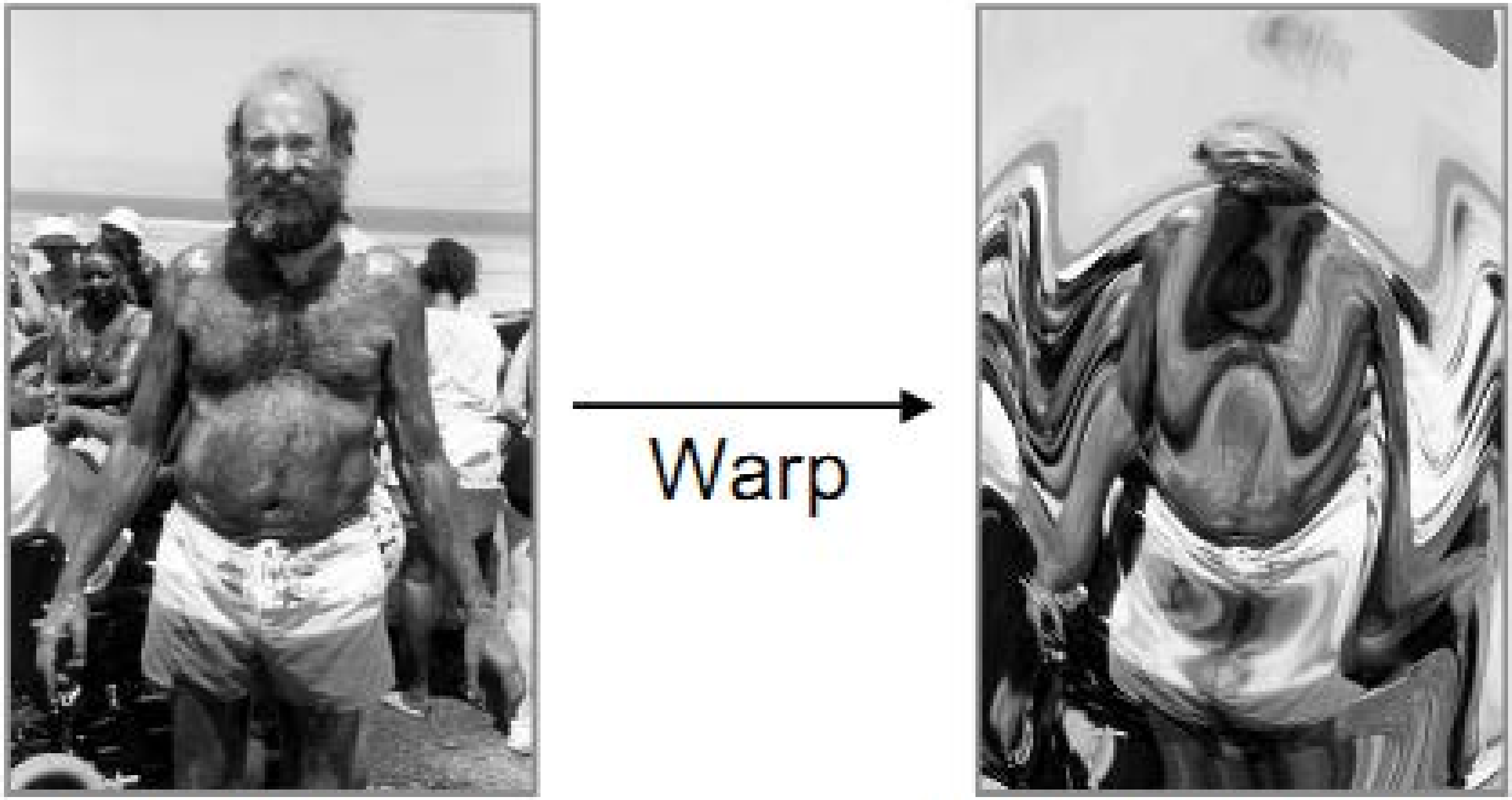
Image Warping



Source image

Destination image

Image Mapping

- Define transformation
  - Describe the destination (x,y) for every location (u,v) in the source (or vice-versa, if invertible)

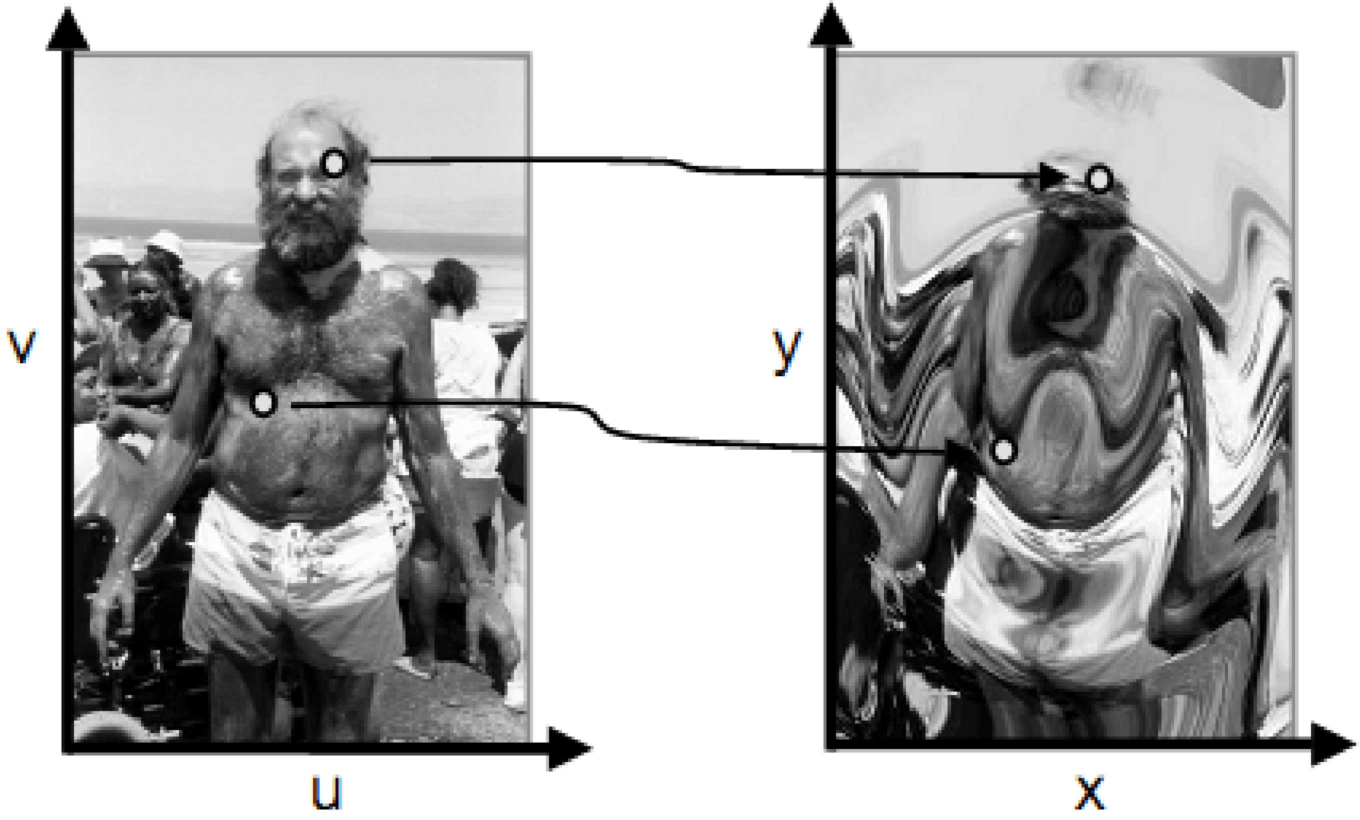
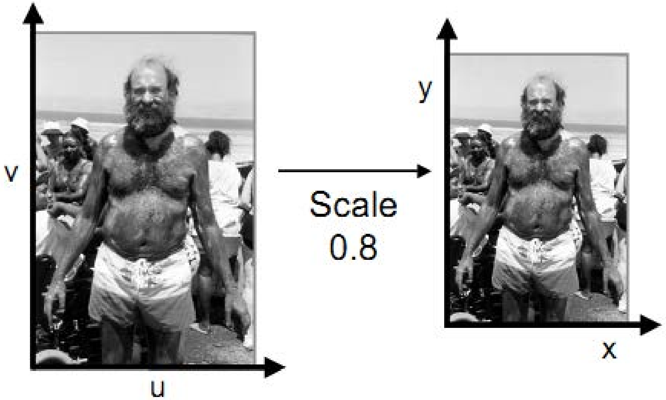
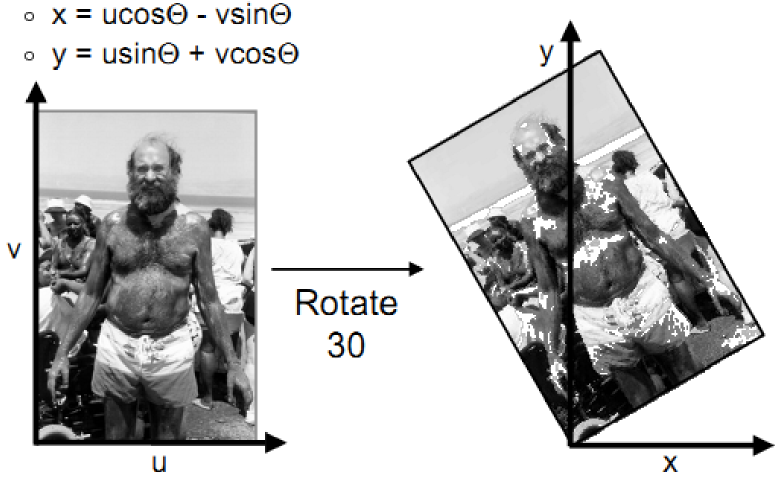


Image Mapping - Examples

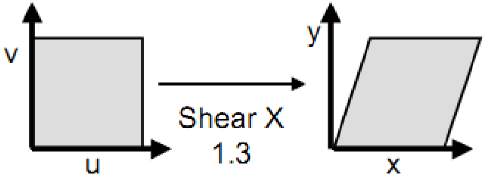
- Scale by *factor*:
  - $x = \mathit{factor} * u$
  - $y = \mathit{factor} * v$



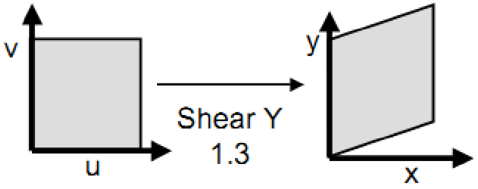
- Rotate by Θ degrees:
  - $x = u\cos\Theta - v\sin\Theta$
  - $y = u\sin\Theta + v\cos\Theta$



- Shear in X by *factor:*
  - $x = u + \mathit{factor} * v$
  - $y = v$



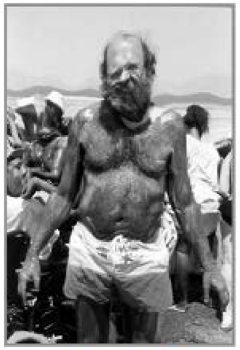
- Shear in Y by *factor:*
  - $x = u$
  - $y = v + \mathit{factor} * u$



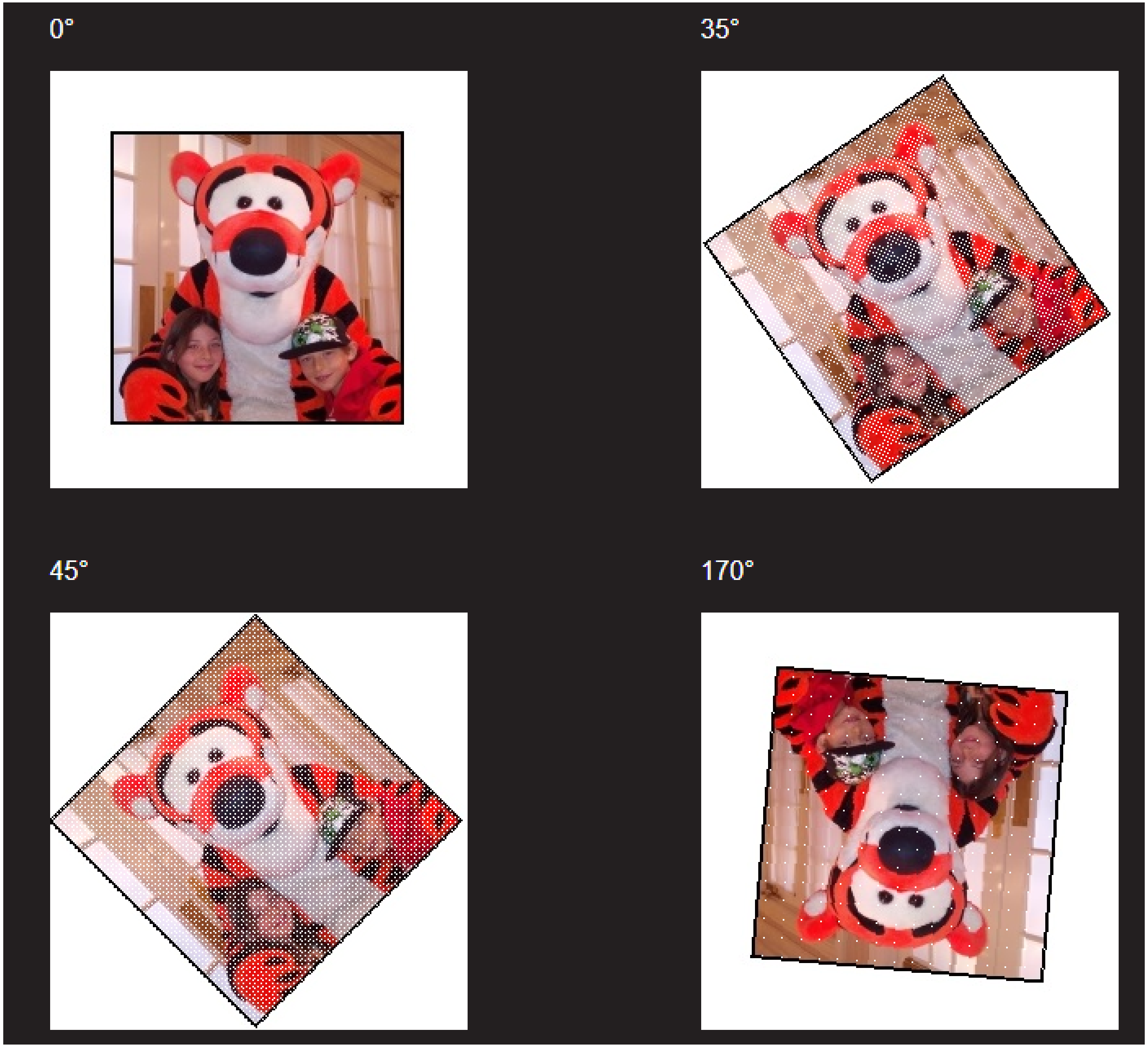
- Any function of u and v:
  - $x = f_x(u,v)$
  - $y = f_y(u,v)$

Fish-eye

"Swirl"

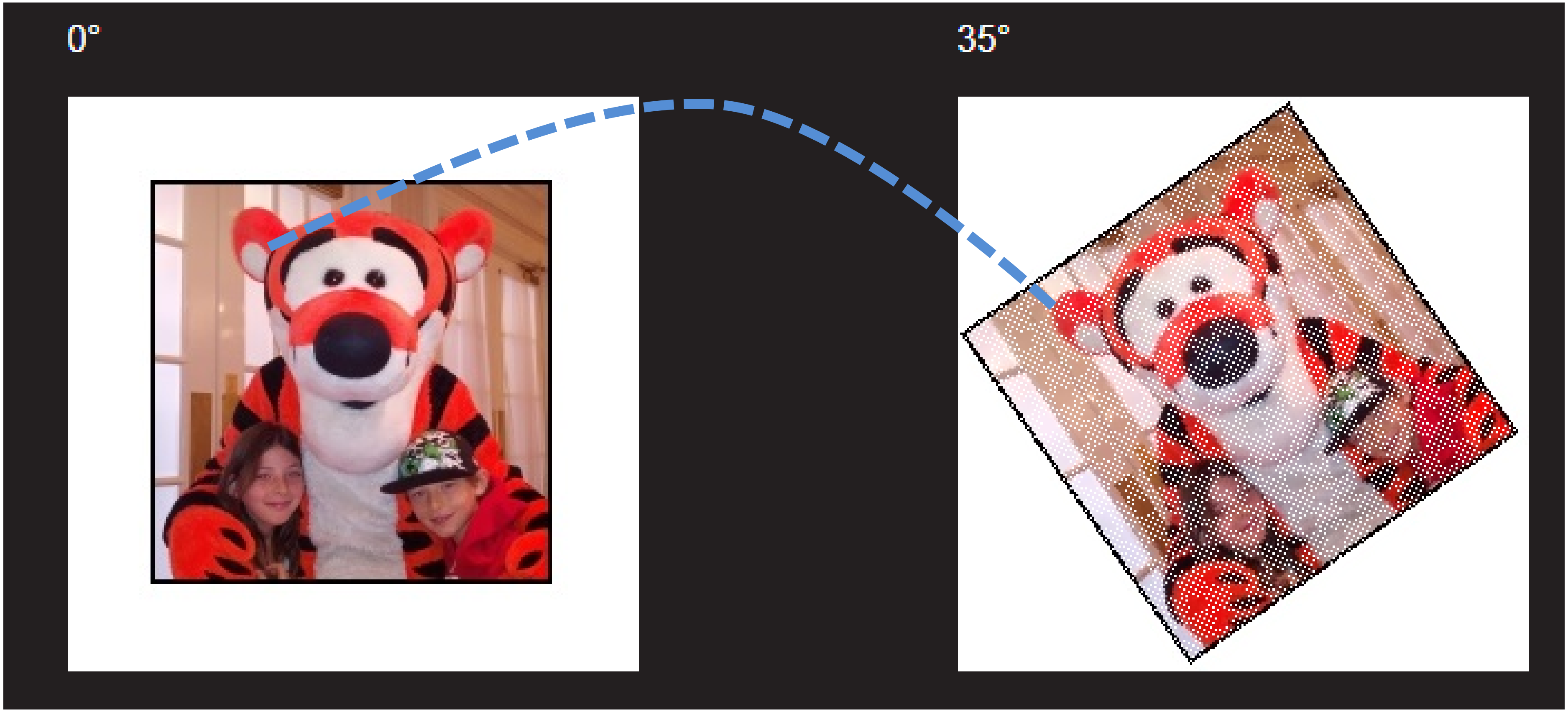


"Rain"


0°
35°
45°
170°

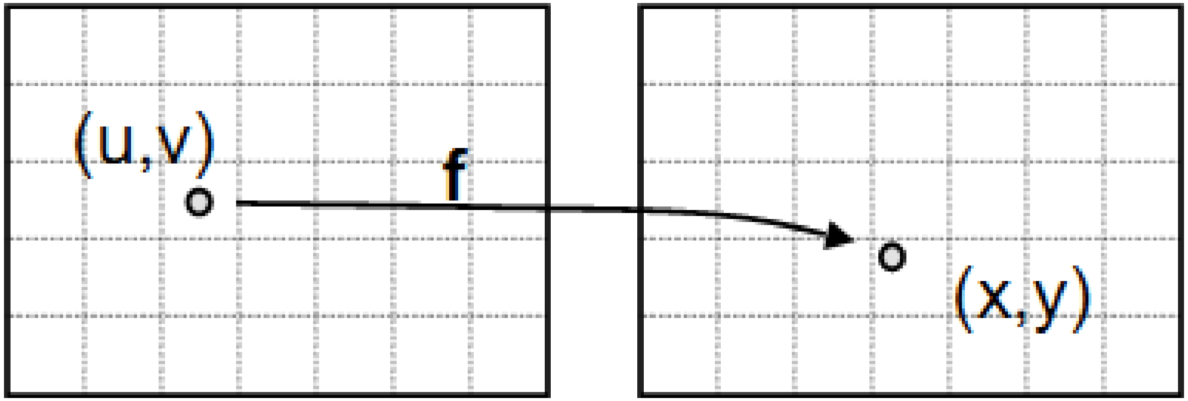
$$\begin{bmatrix} x^* \\ y^* \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & -\tan(\theta/2) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \sin\theta & 1 \end{bmatrix} \begin{bmatrix} 1 & -\tan(\theta/2) \\ 0 & 1 \end{bmatrix}}_{\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}} \begin{bmatrix} x \\ y \end{bmatrix}$$

Forward Mapping



Forward Mapping

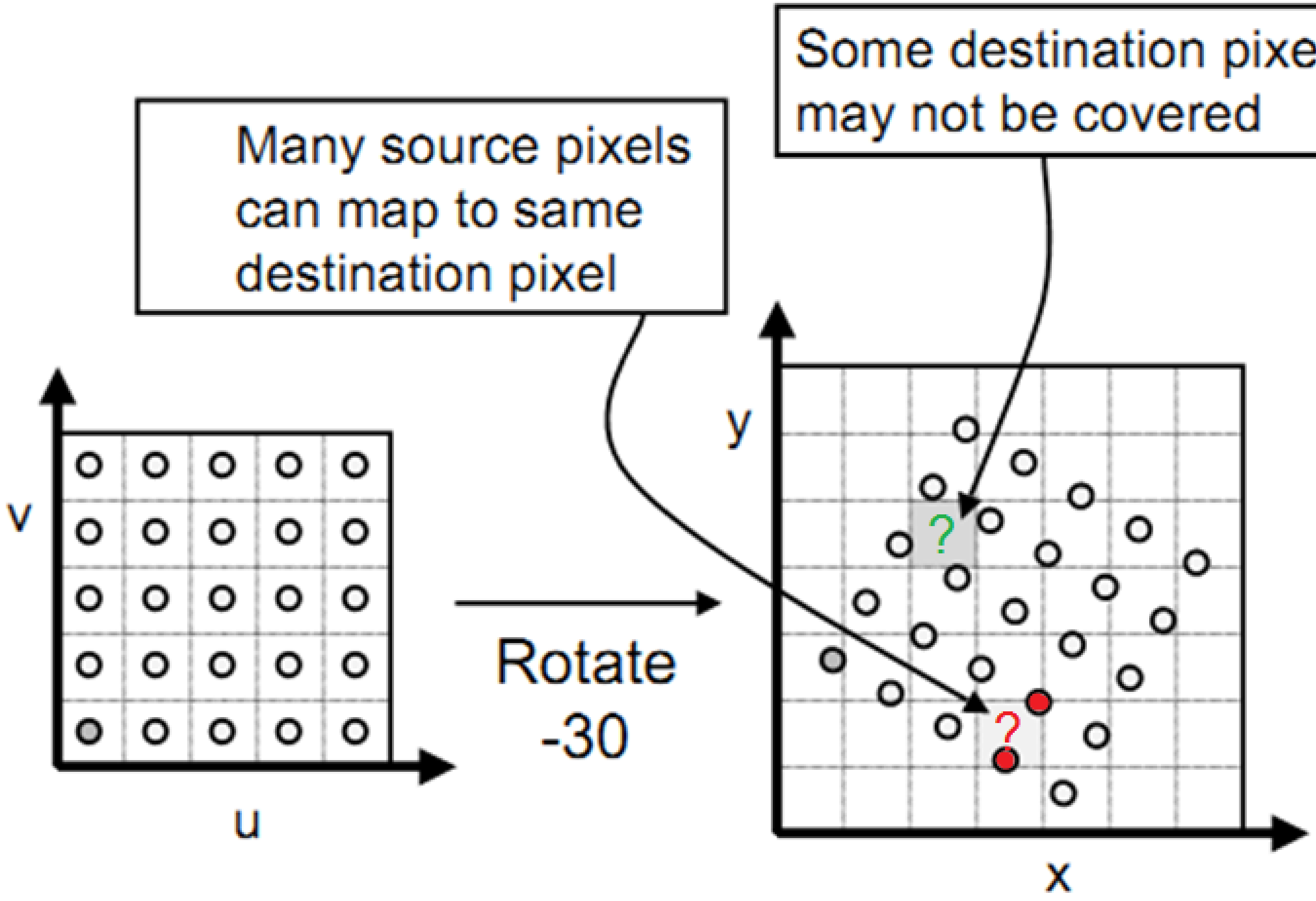
```
for (int u = 0; u < umax; u++) {
  for (int v = 0; v < vmax; v++) {
    float x = $f_x$(u,v);
    float y = $f_y$(u,v);
    dst(x,y) = src(u,v);
  }
}
```



Source image

Destination image

Forward Mapping - Disadvantages

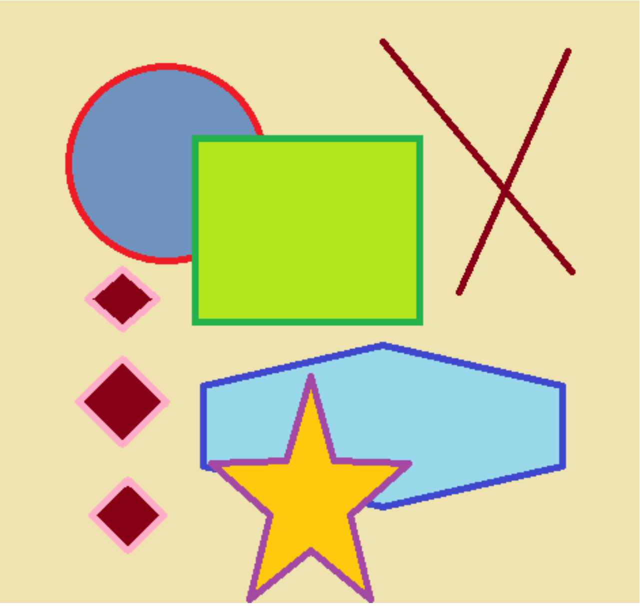


Many source pixels can map to same destination pixel

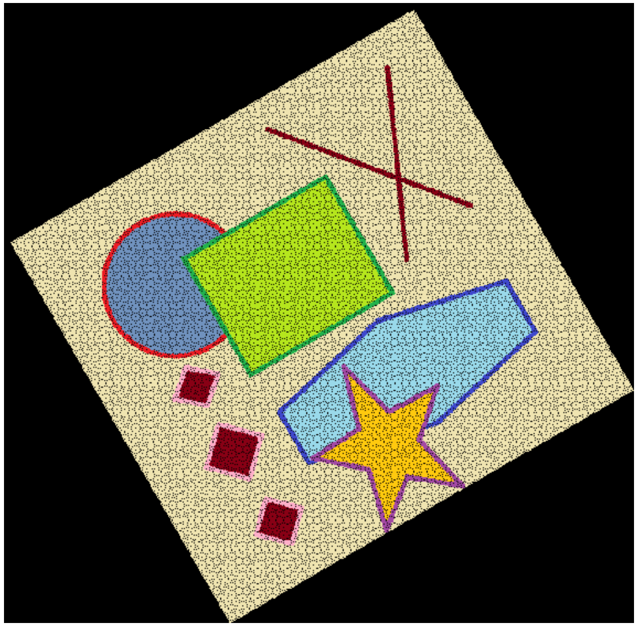
Some destination pixels may not be covered

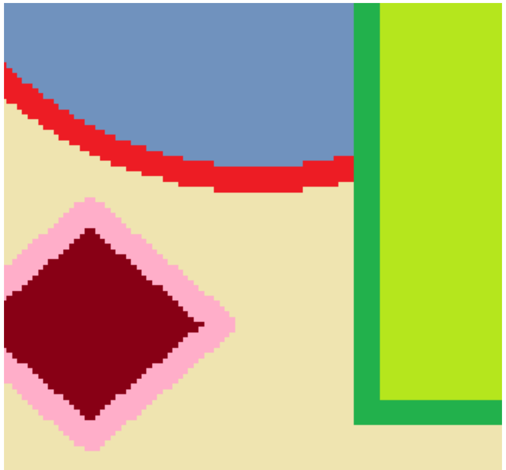
Example – Forward Mapping

Original

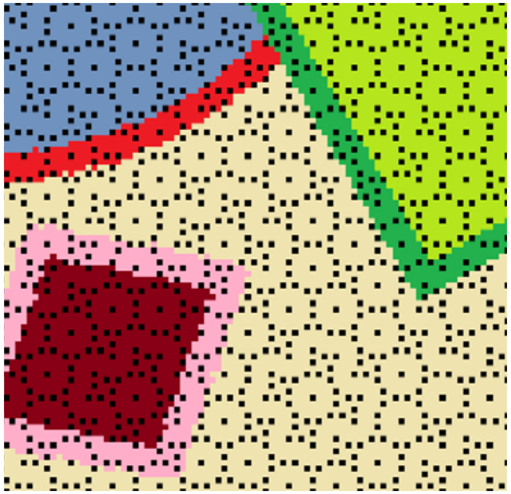


Rotated





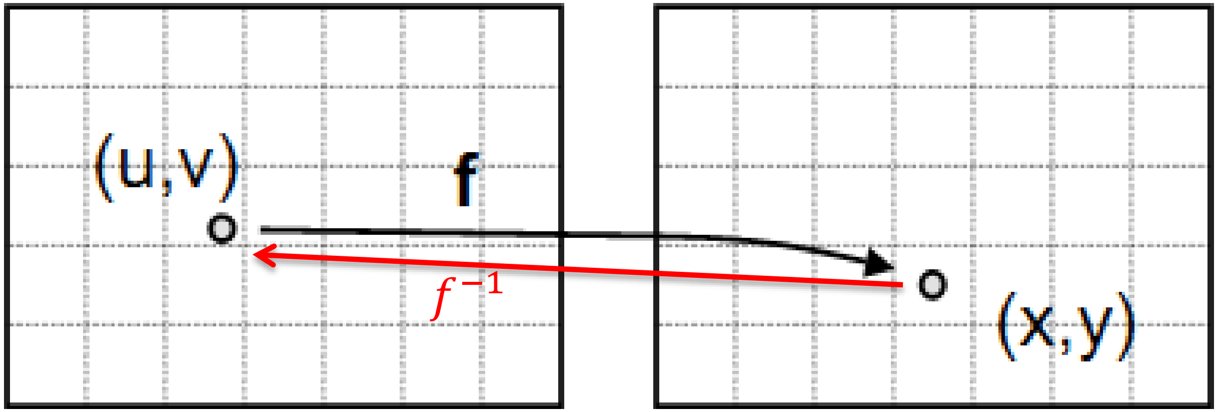
Zoom In



Backward Mapping

```
for (int x = 0; x < xmax; x++) {
   for (int y = 0; y < ymax; y++) {
      float u = $f_x^{-1}(x,y)$;
      float v = $f_y^{-1}(x,y)$;
      dst(x,y) = src(u,v);
   }
}
```

The Problem:
(u,v) are not integers!



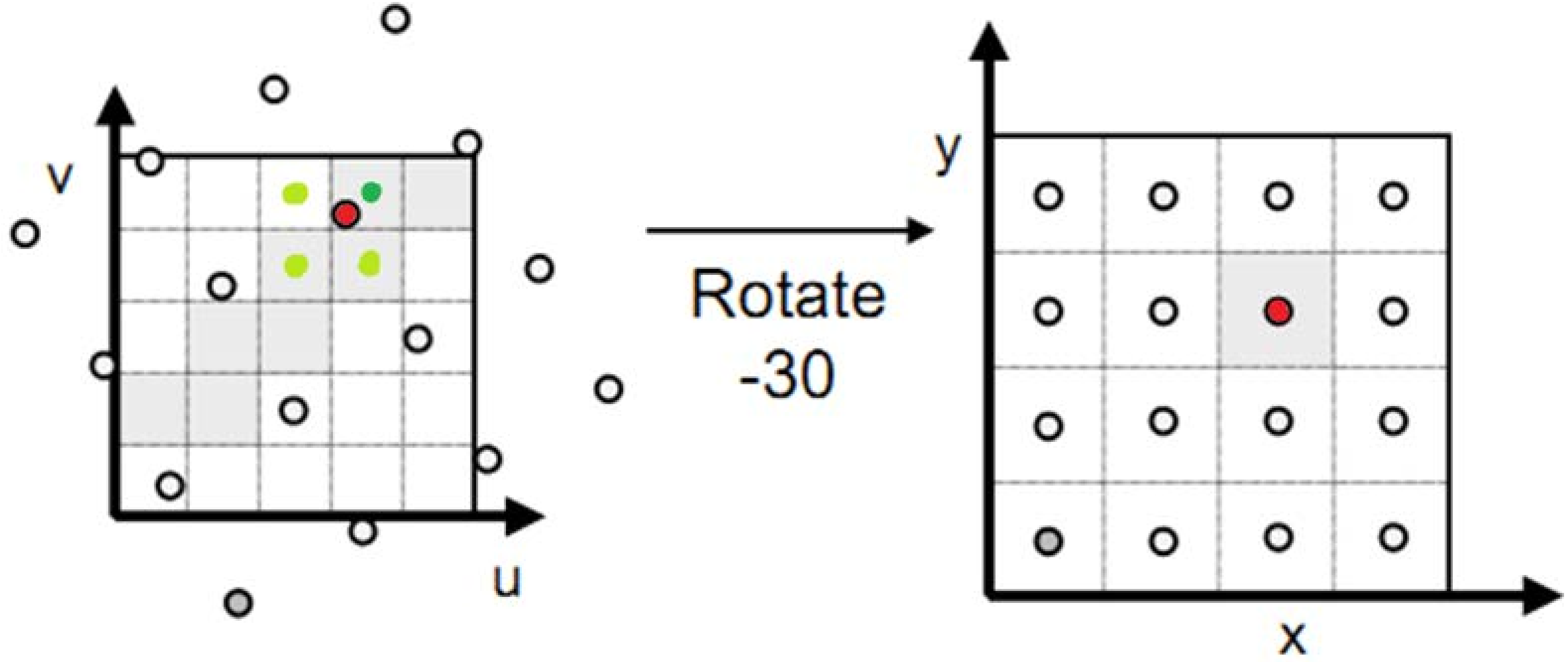
Source image

Destination image

Nearest Neighbor

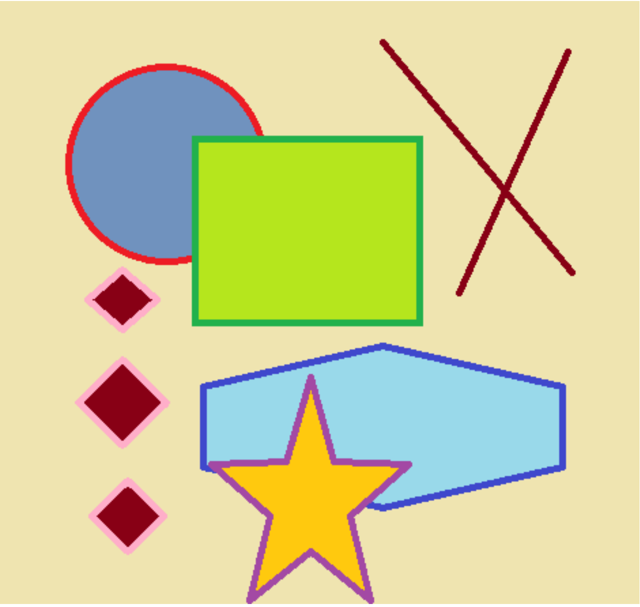
- Take value at closest pixel:
  - int iu = trunc(u+0.5);
  - int iv = trunc(v+0.5);
  - dst(x,y) = src(iu,iv);

This method is simple, but it causes aliasing

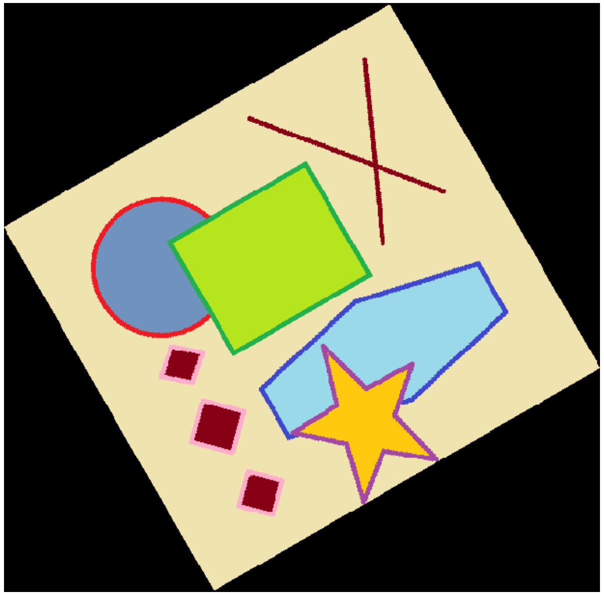


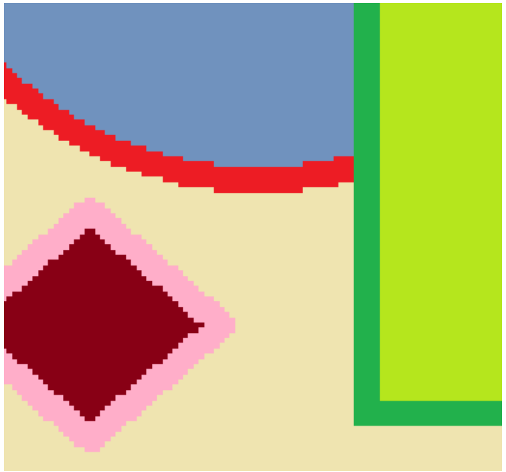
Example - Nearest Neighbor

Original

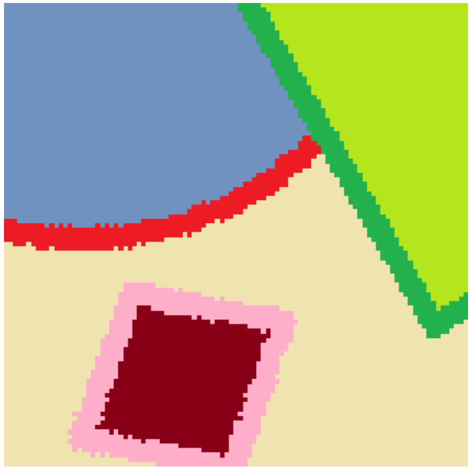


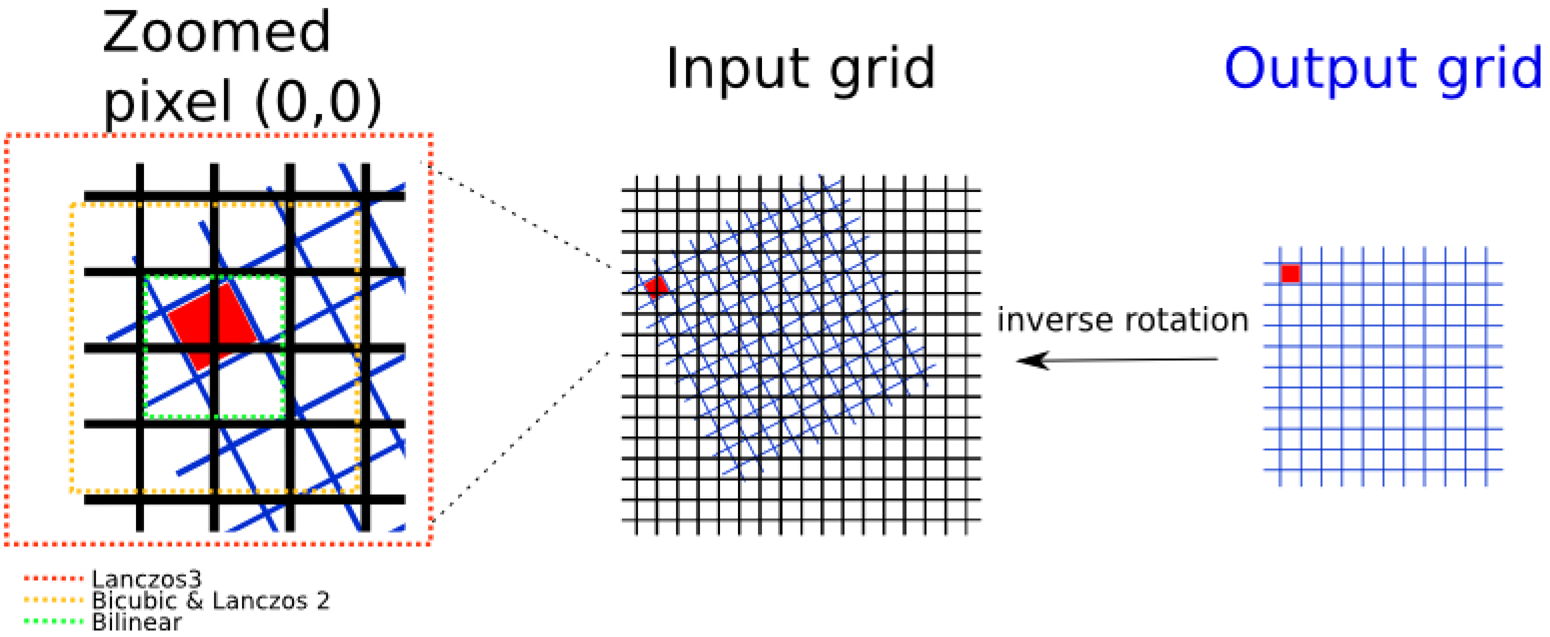
Rotated





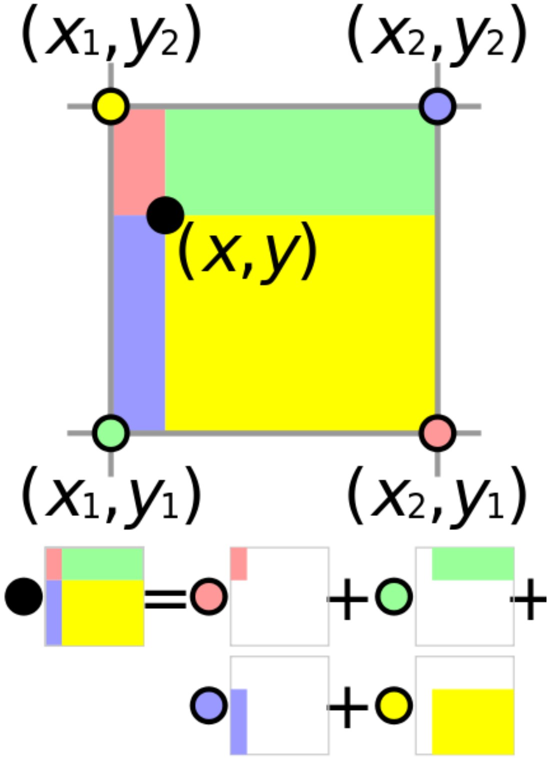
Zoom In



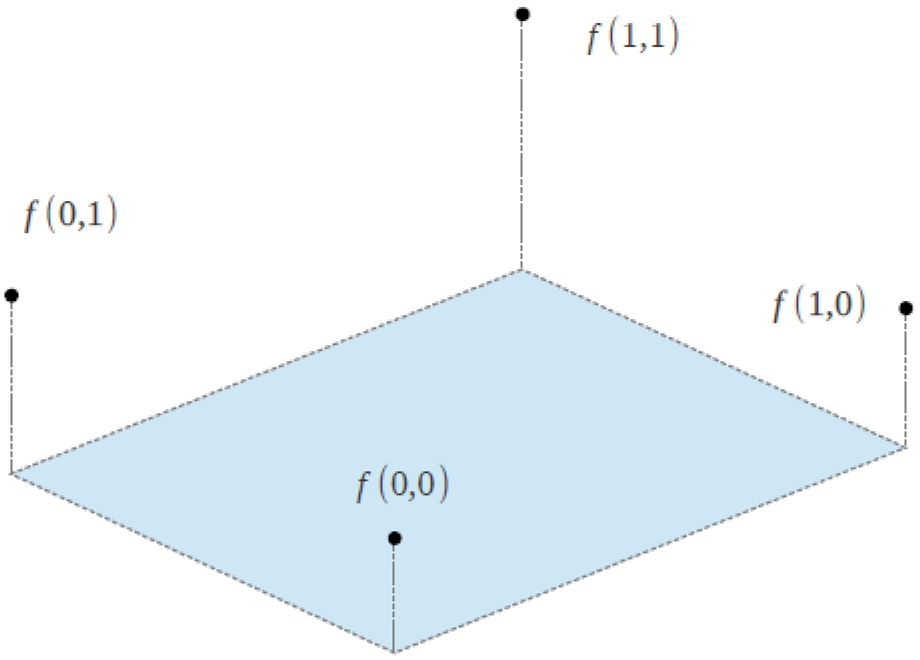

Zoomed pixel (0,0)
Input grid
Output grid
inverse rotation
Lanczos3
Bicubic & Lanczos 2
Bilinear

Bi-linear

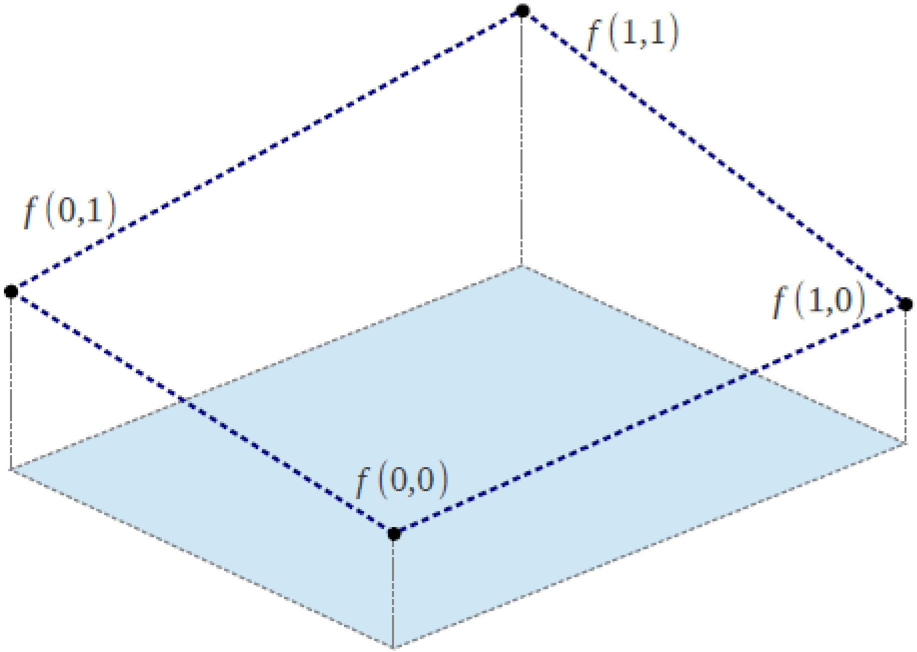
- Bi-linear interpolates four closest pixels.
- The weight for each pixel is proportional to its distance from the sampling point (x,y)



Bi-linear Interpolation

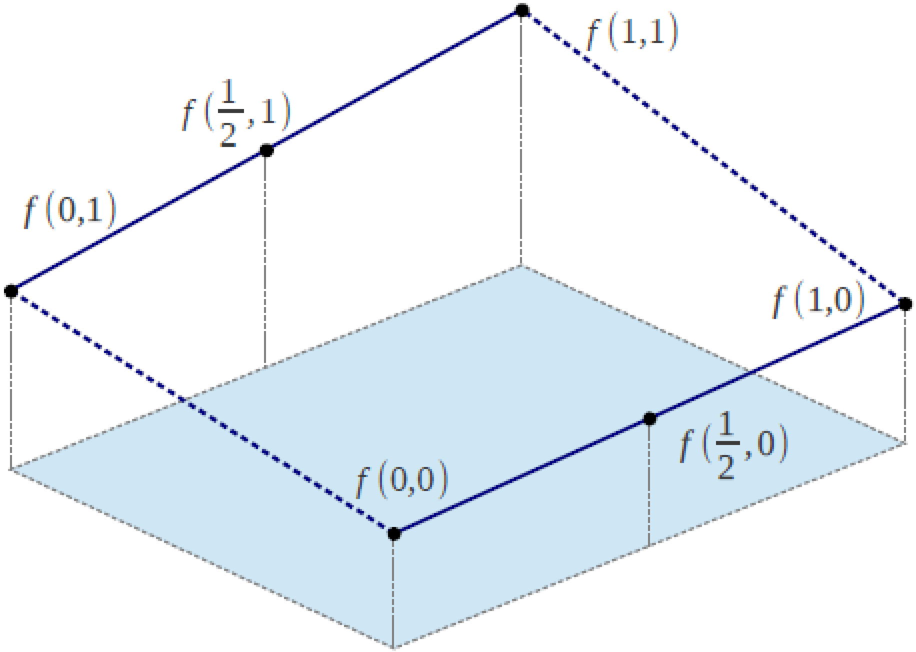


Bi-linear Interpolation



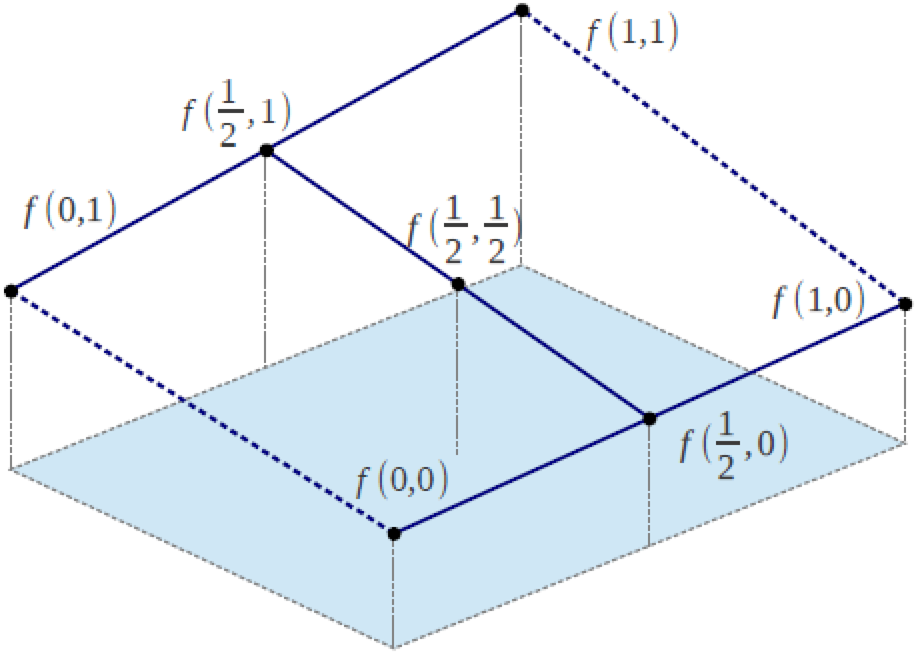
- Model $f(x, y)$ as a bilinear surface

Bi-linear Interpolation



- Model $f(x,y)$ as a bilinear surface
- Interpolate $f(\frac{1}{2},0)$ using $f(0,0)$ and $f(1,0)$
  Interpolate $f(\frac{1}{2},1)$ using $f(0,1)$ and $f(1,1)$

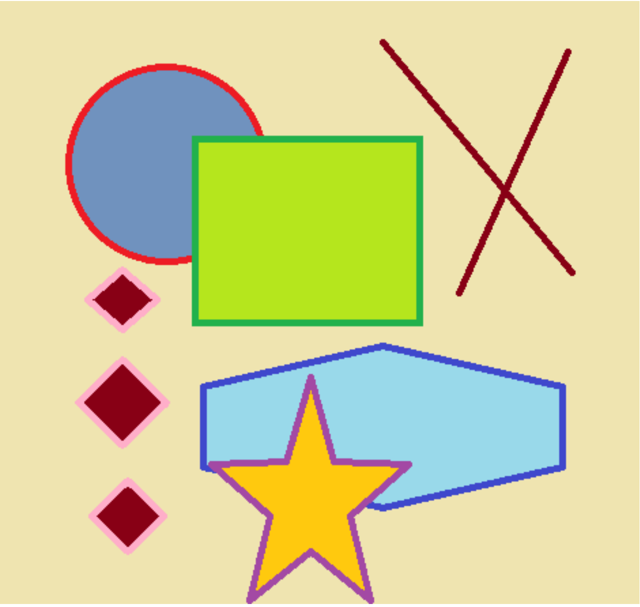
Bi-linear Interpolation



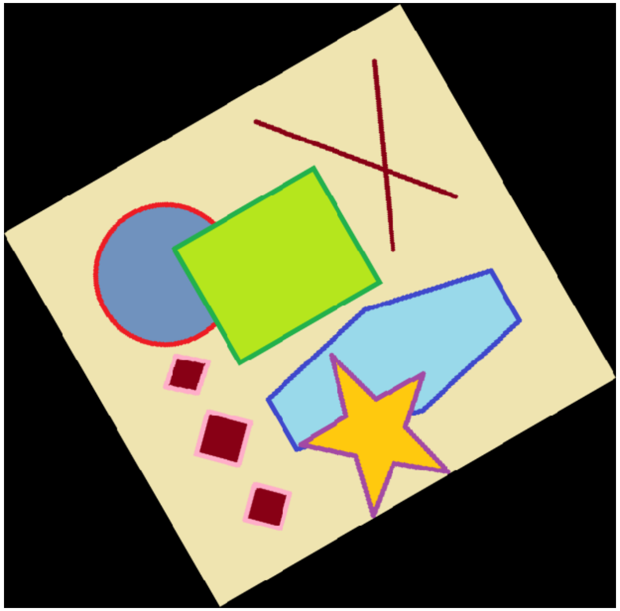
- Model $f(x,y)$ as a bilinear surface
- Interpolate $f(\frac{1}{2},0)$ using $f(0,0)$ and $f(1,0)$
  Interpolate $f(\frac{1}{2},1)$ using $f(0,1)$ and $f(1,1)$
- Interpolate $f(\frac{1}{2},\frac{1}{2})$ using $f(\frac{1}{2},0)$ and $f(\frac{1}{2},1)$

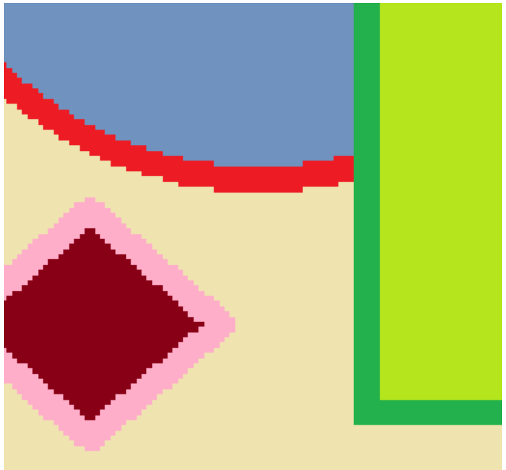
Example Bi-linear

Original

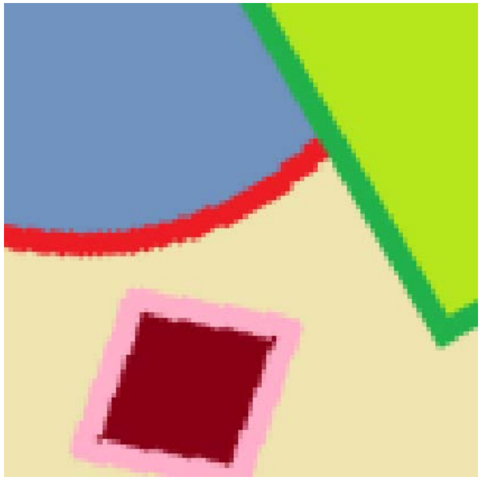


Rotated



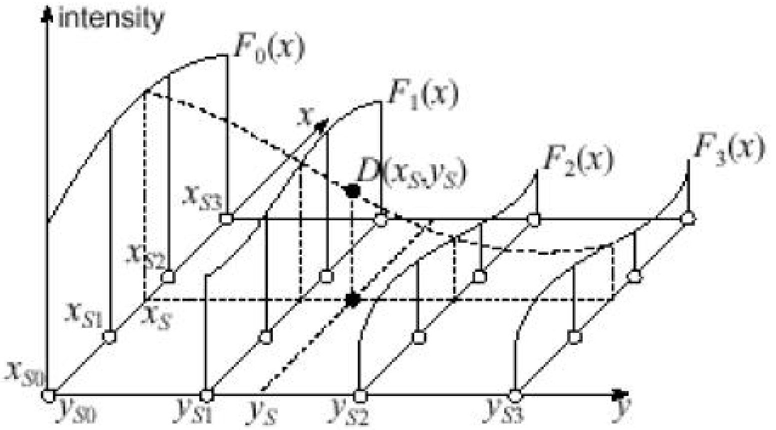


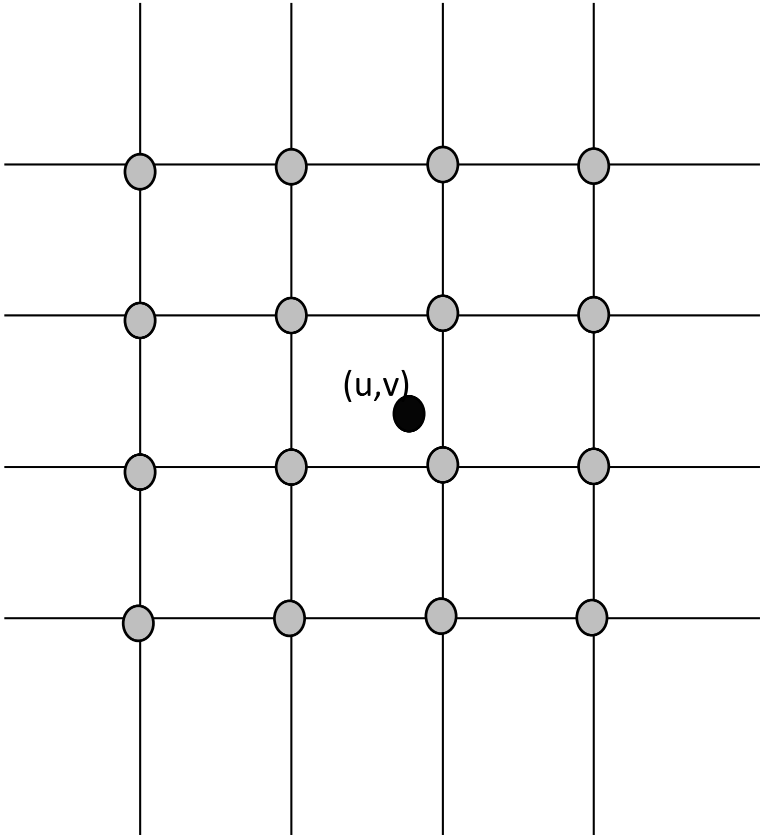
Zoom In



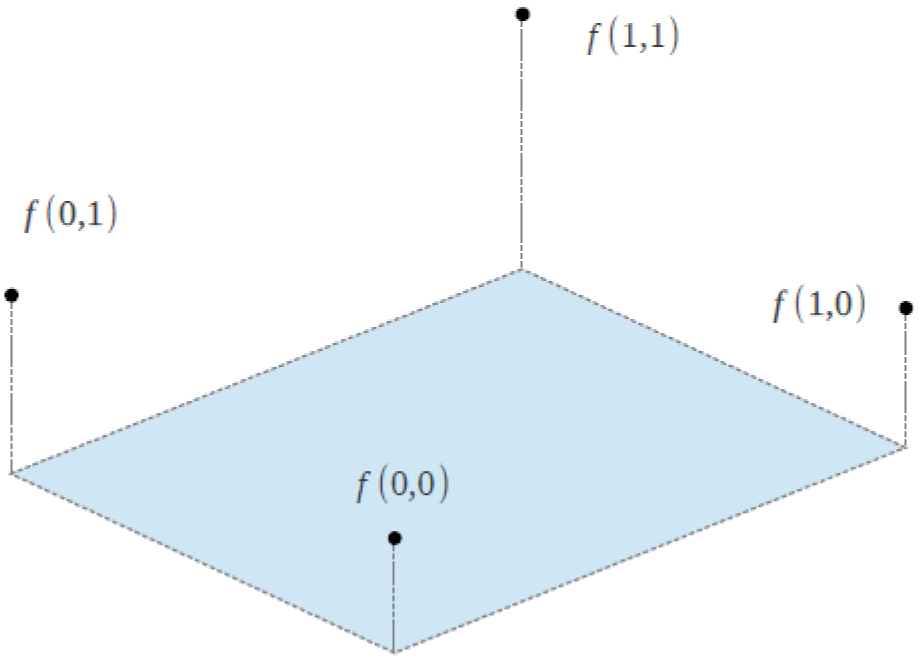
Bi-cubic

- Bicubic interpolates 16 closest neighbors (4x4 neighborhood)
  - The result is much more smooth

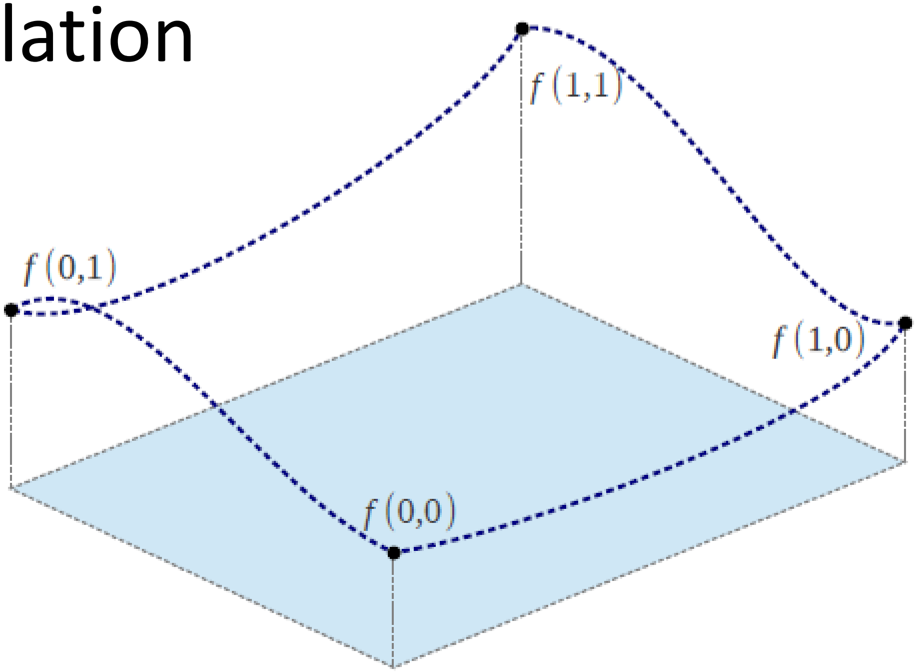




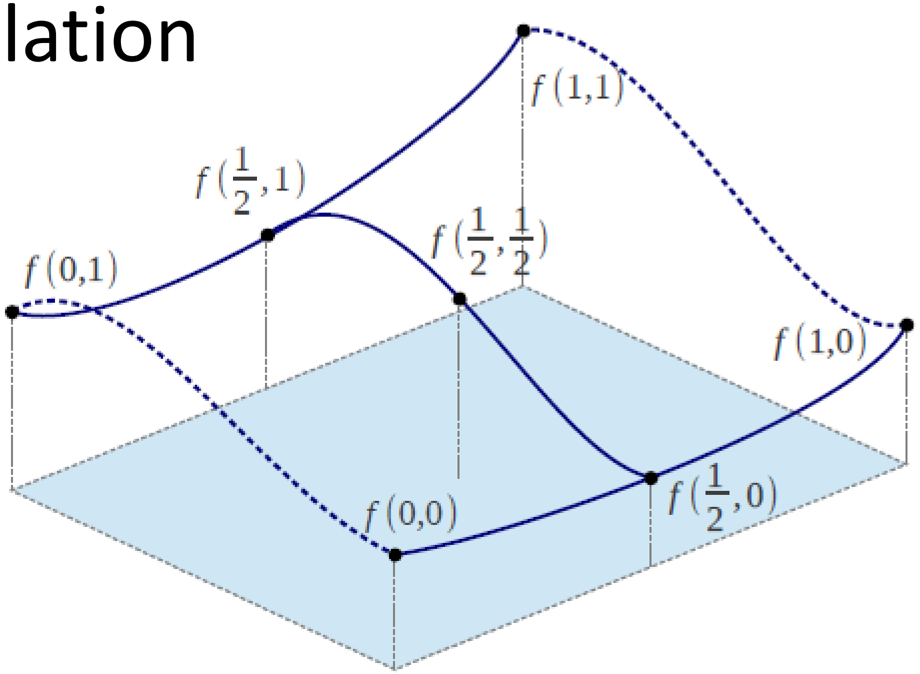
Bi-cubic Interpolation



Bi-cubic Interpolation



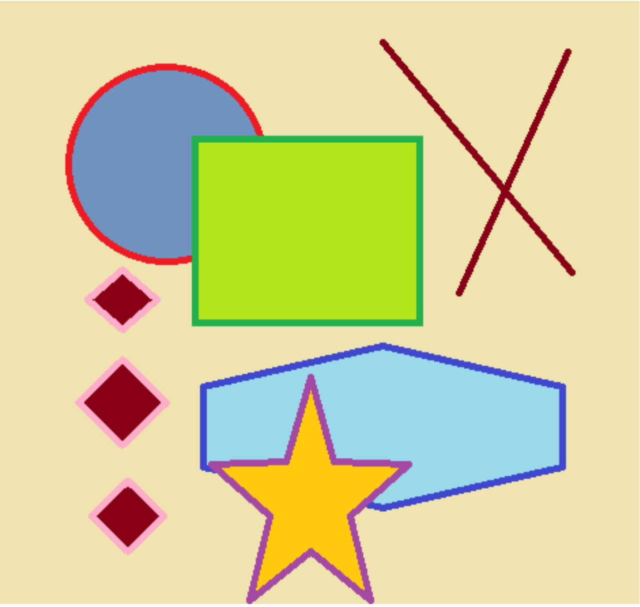
Bi-cubic Interpolation



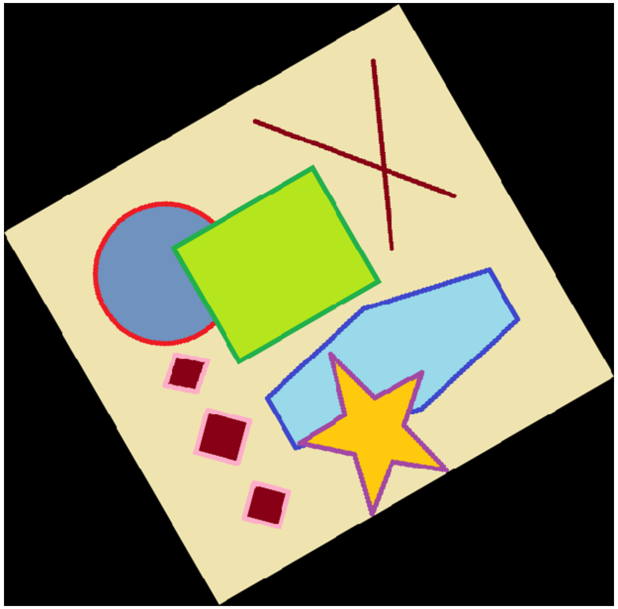
- Interpolate
  - $f(\frac{1}{2},0)$ using $f(0,0)$, $f(1,0)$, $\partial_x f(0,0)$ and $\partial_x f(1,0)$
  - $f(\frac{1}{2},1)$ using $f(0,1)$, $f(1,1)$, $\partial_x f(0,1)$ and $\partial_x f(1,1)$
  - $\partial_y f(\frac{1}{2},0)$ using $\partial_y f(0,0)$, $\partial_y f(1,0)$, $\partial_{xy} f(0,0)$ and $\partial_{xy} f(1,0)$
  - $\partial_y f(\frac{1}{2},1)$ using $\partial_y f(0,1)$, $\partial_y f(1,1)$, $\partial_{xy} f(0,1)$ and $\partial_{xy} f(1,1)$
- Interpolate $f(\frac{1}{2},\frac{1}{2})$ using $f(\frac{1}{2},0)$, $f(\frac{1}{2},1)$, $\partial_y f(\frac{1}{2},0)$ and $\partial_y f(\frac{1}{2},1)$

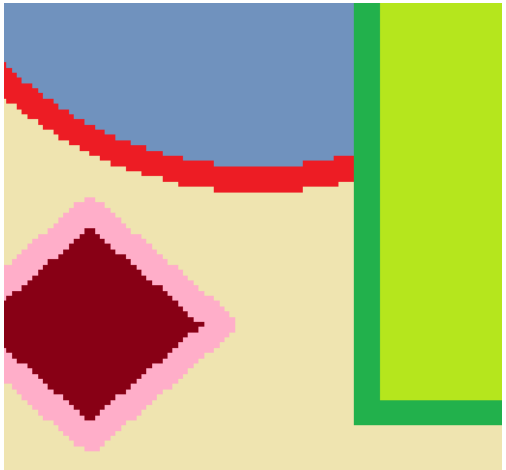
Example Bi-cubic

Original

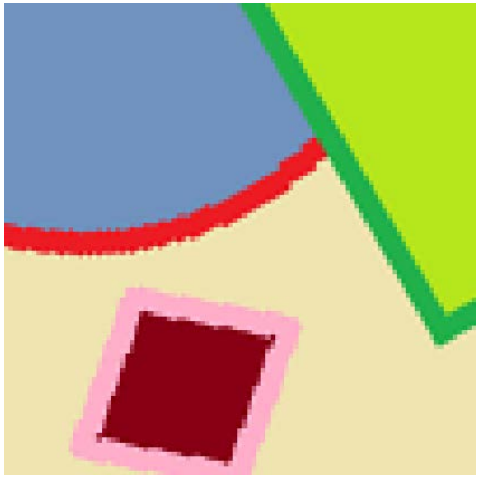


Rotated

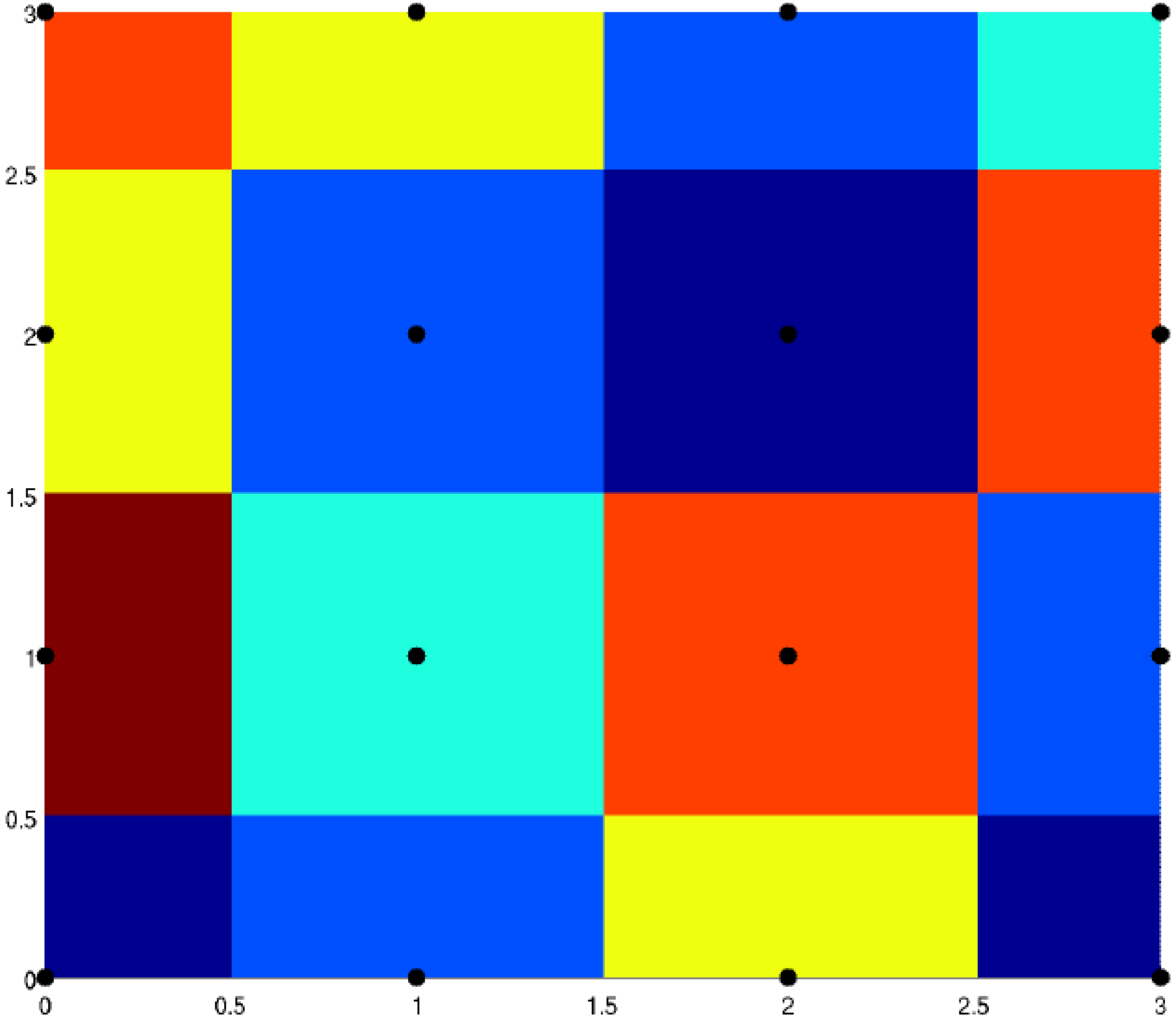




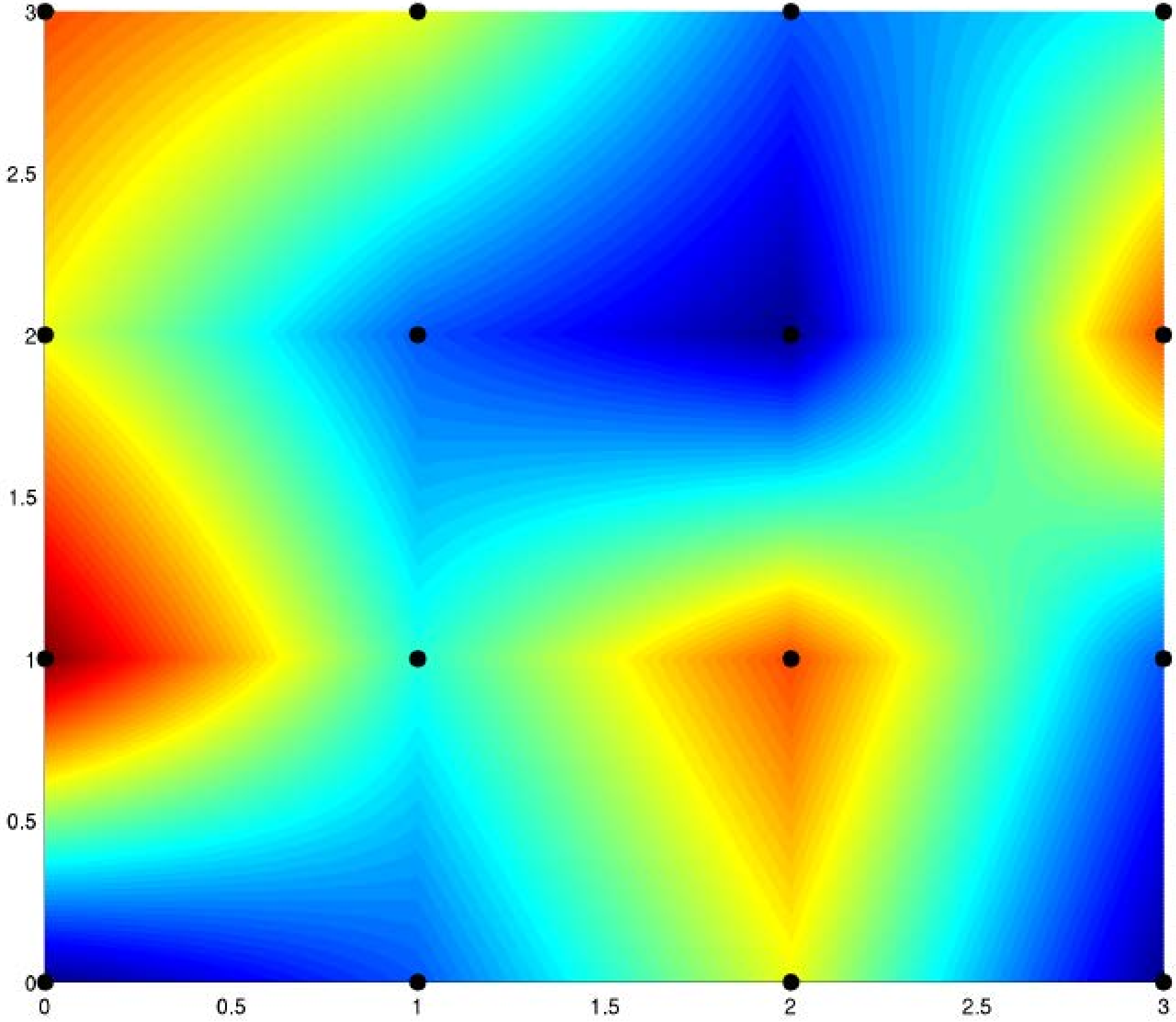
Zoom In



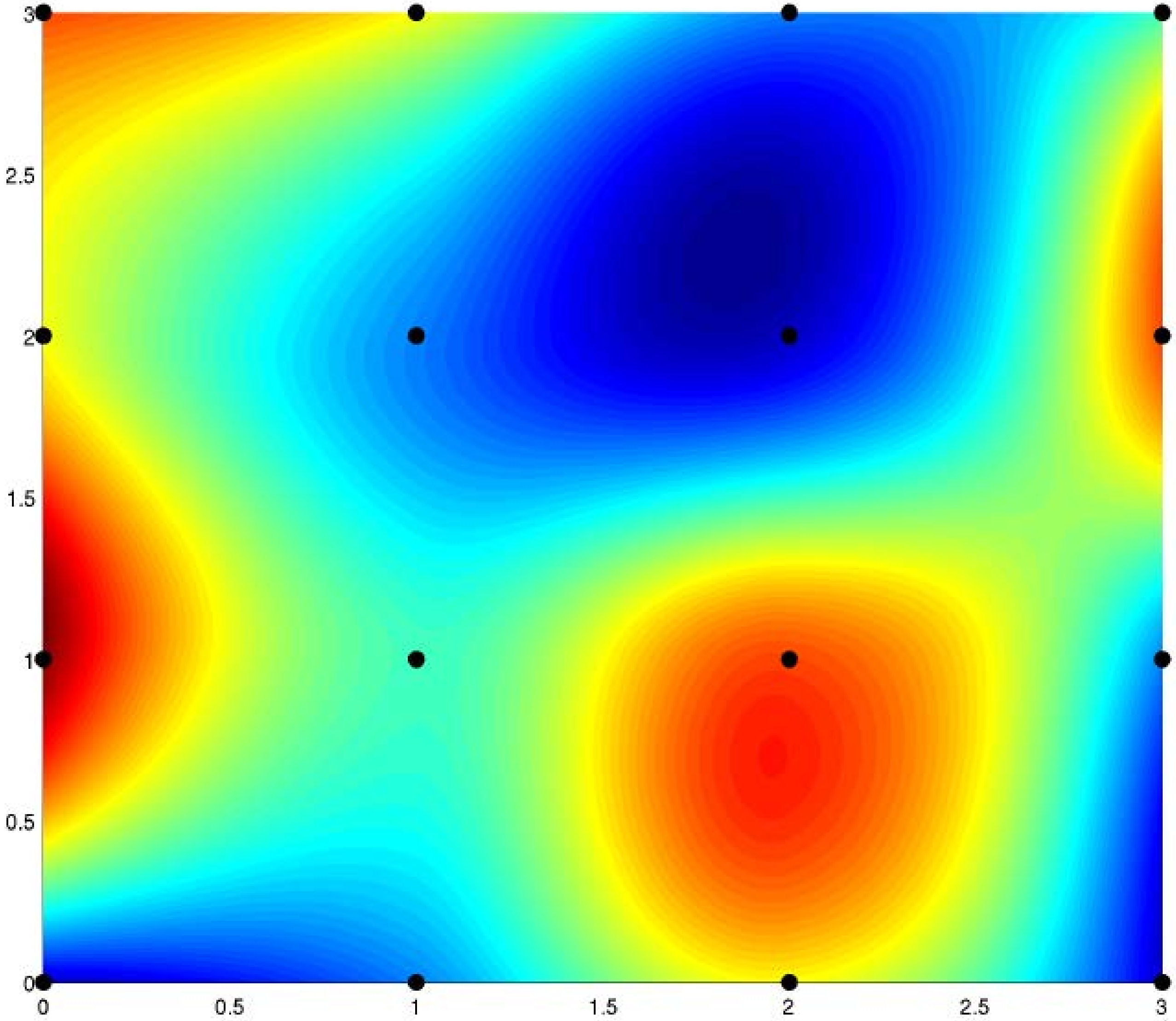
Nearest Neighbor



Bi-Linear Interpolation

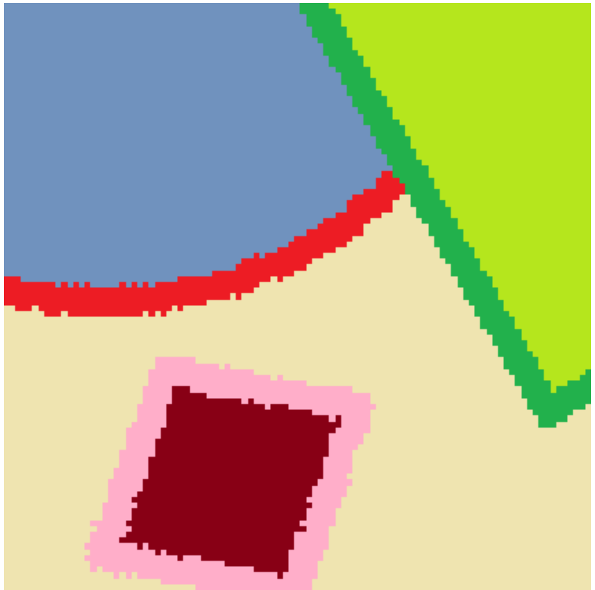


Bi-Cubic Interpolation

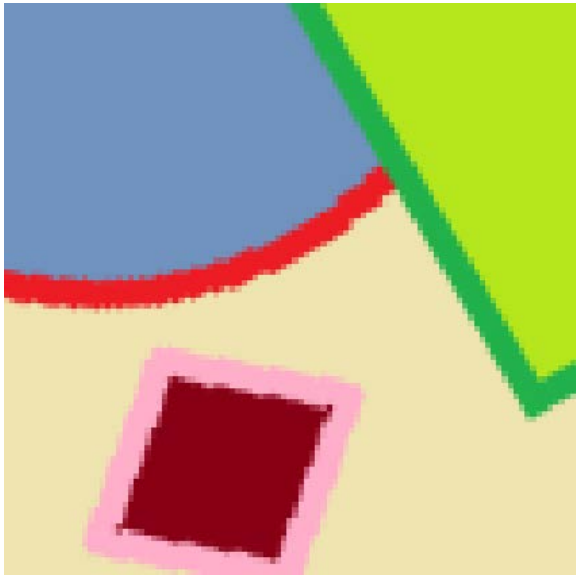


Comparison

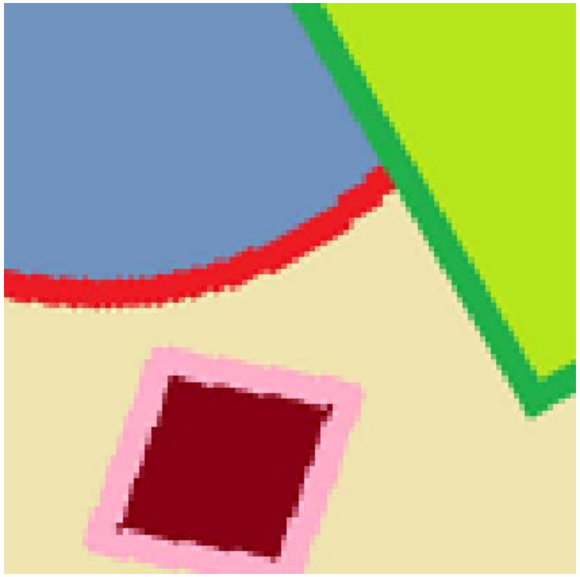
Nearest Neighbor



Bi-linear



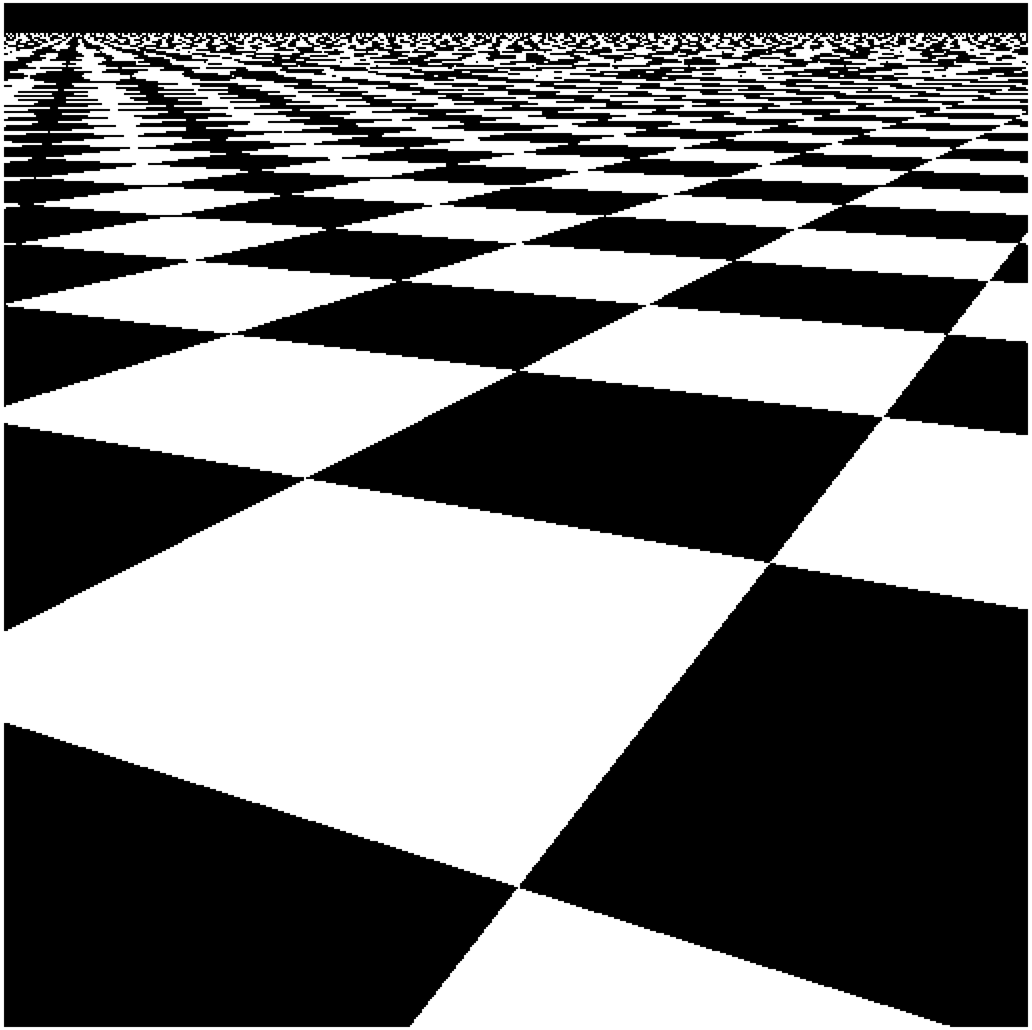
Bi-cubic



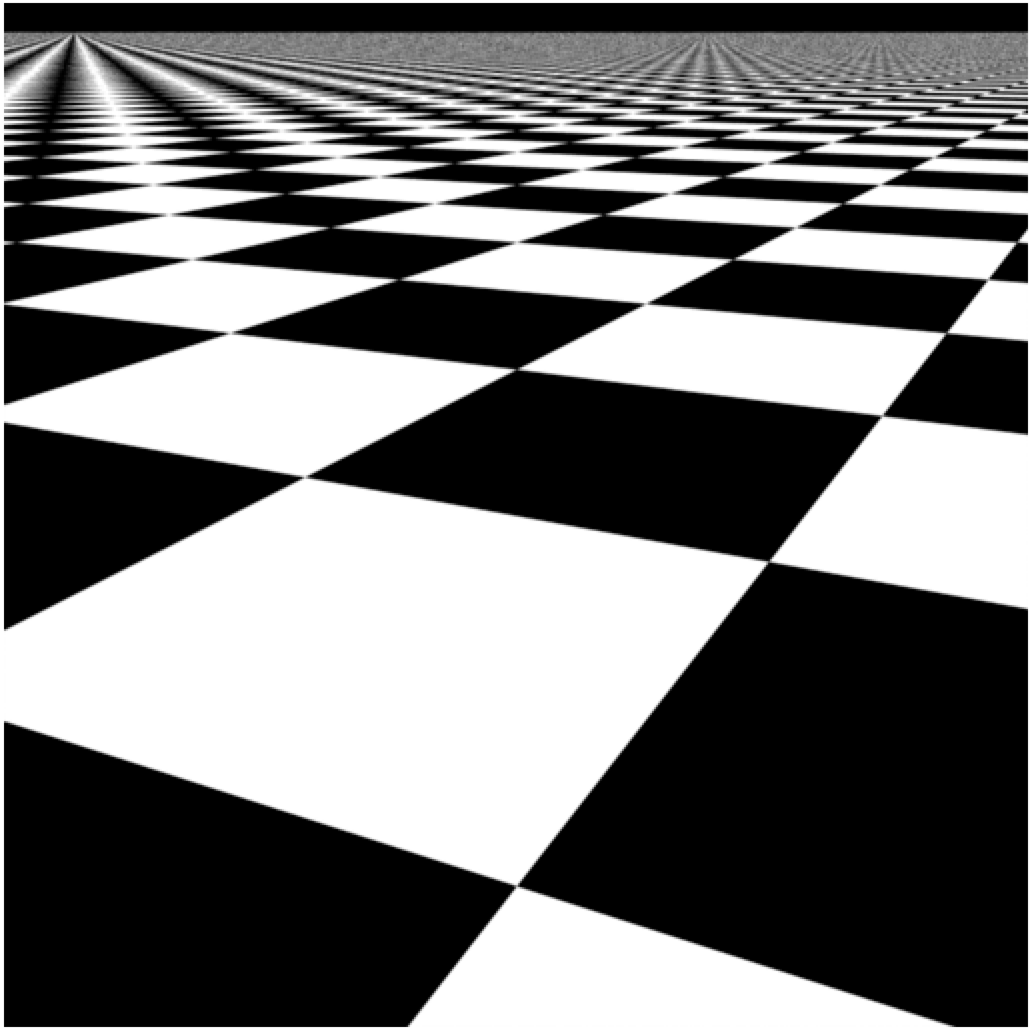

UNITED STATES OF AMERICA
E PLURIBUS UNUM
HALF DOLLAR


UNITED STATES OF AMERICA
E PLURIBUS UNUM
HALF DOLLAR

Nearest neighbor sampling

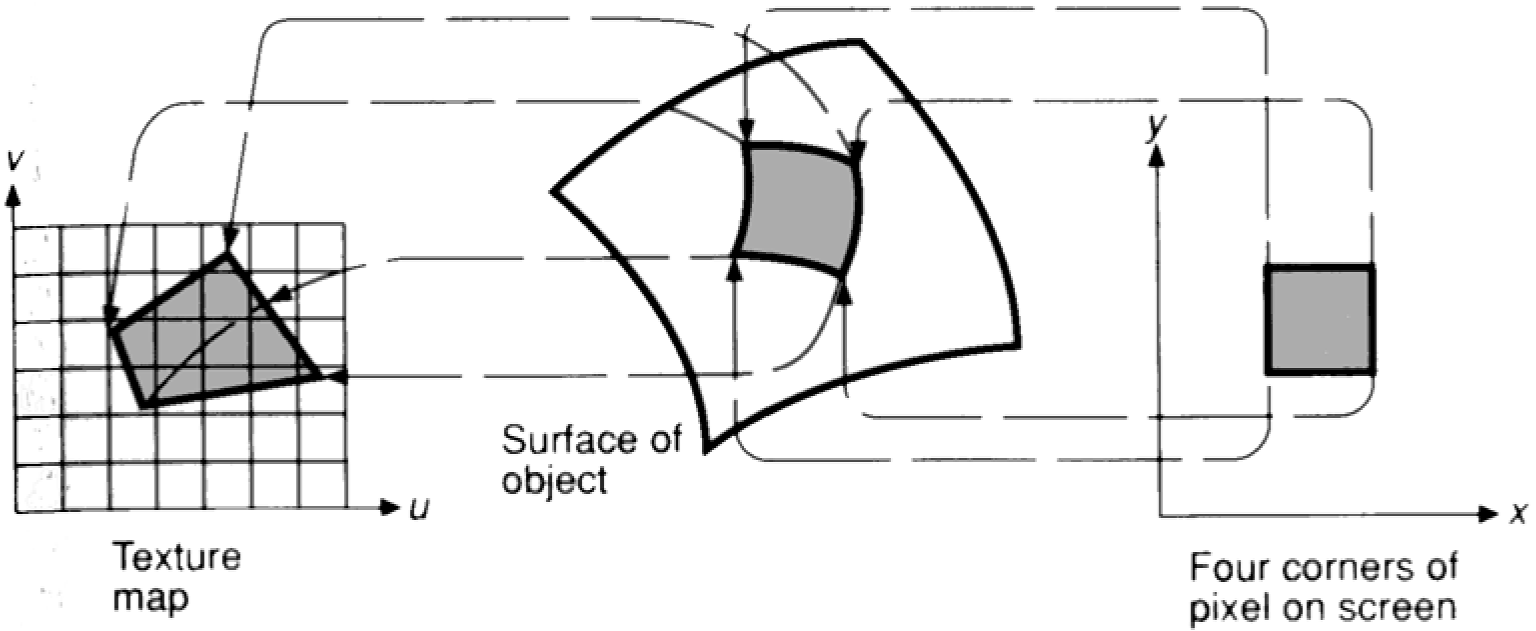


Filtered Texture:

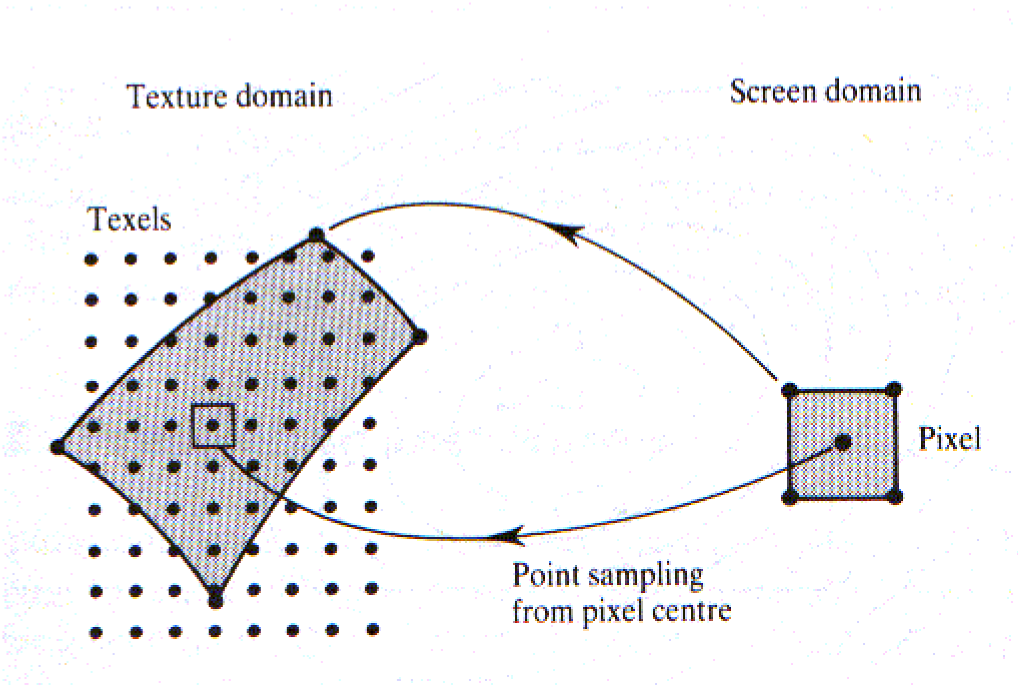


Texture Aliasing

- A single screen space pixel might correspond to many texels (texture elements):



Texture Mapping



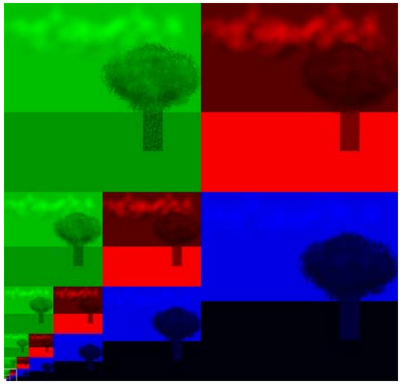
Texture Pre-Filtering

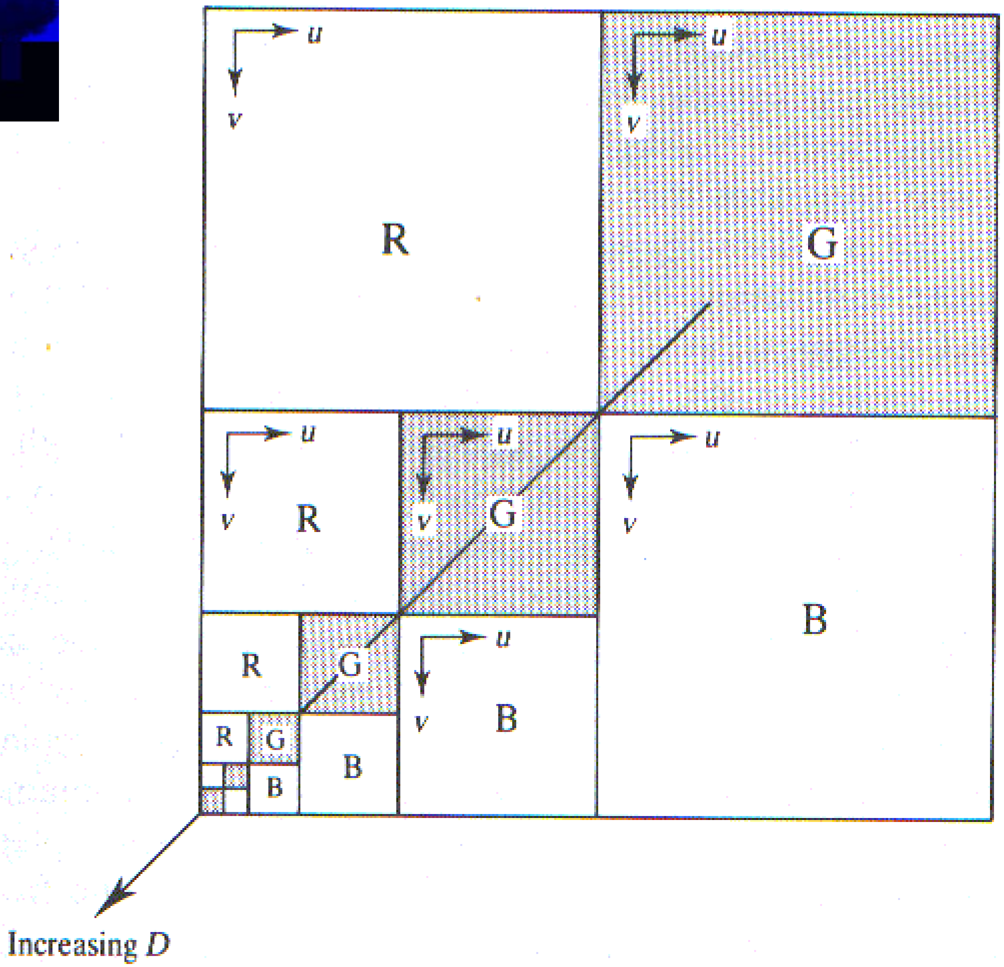
- Problem: filtering the texture during rendering is too slow for interactive performance.
- Solution: pre-filter the texture in advance
  - Summed area tables - gives the average value of each axis-aligned rectangle in texture space
  - Mip-maps (tri-linear interpolation) - supported by most of today’s texture mapping hardware

MIP-Maps

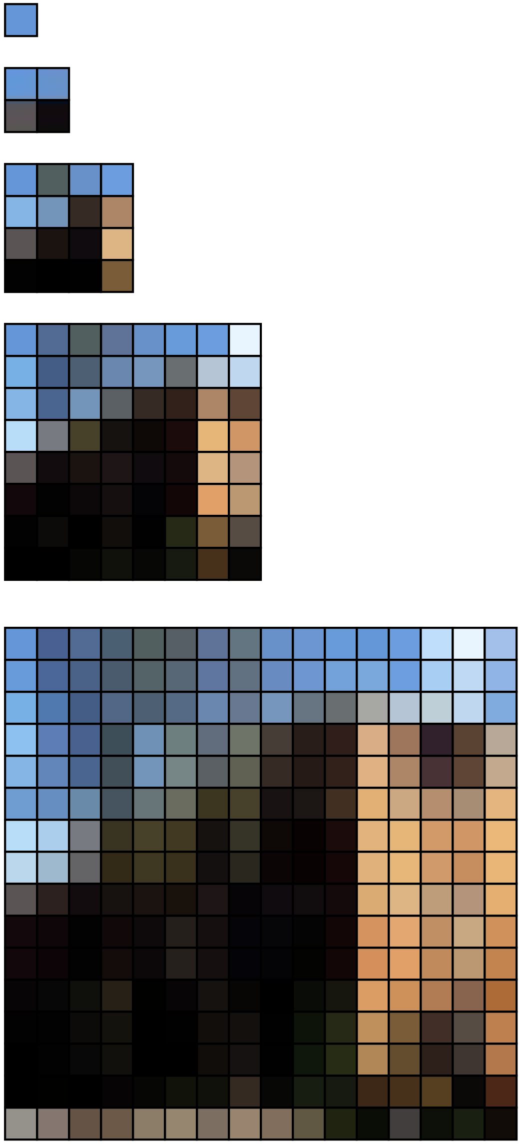
- Precompute a set of prefiltered textures (essentially an image pyramid).
- Based on the area of the pre-image of the pixel:
  - Select two “best” resolution levels
  - Use bilinear interpolation inside each level
  - Linearly interpolate the results
- Referred to as trilinear interpolation

MIP Maps





MIP Mapping



- Lance Williams, 1983
- Create a resolution pyramid of textures
  - Repeatedly subsample texture at half resolution
  - Until single pixel
  - Need extra storage space

- Accessing
  - Use texture resolution closest to screen resolution
  - Or interpolate between two closest resolutions

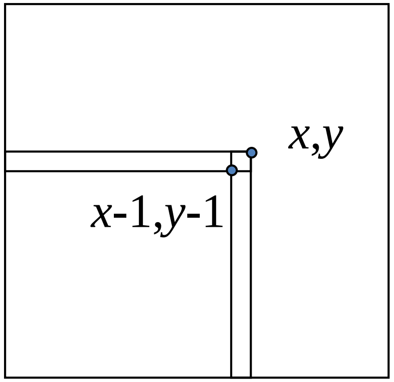
Texture Aliasing

- Image mapped onto polygon
- Occur when screen resolution differs from texture resolution
- Magnification aliasing
  - Screen resolution finer than texture resolution
  - Multiple pixels per texel
- Minification aliasing
  - Screen resolution coarser than texture resolution
  - Multiple texels per pixel

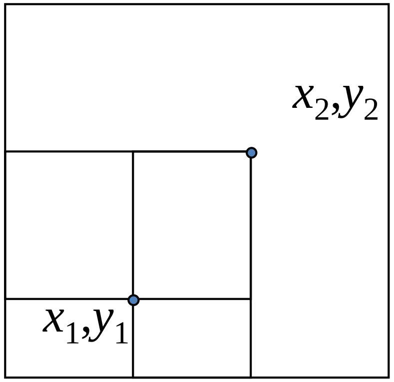
Minification Filtering

- Multiple texels per pixel
- Potential for aliasing since texture signal bandwidth greater than image
- Box filtering requires averaging of texels
- Precomputation
  - MIP Mapping
  - Summed Area Tables

Summed Area Table



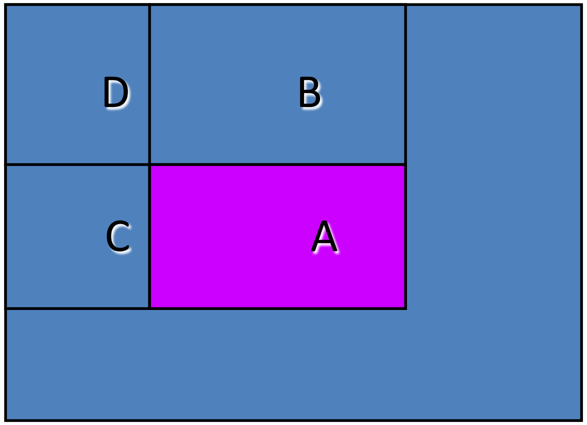
- Frank Crow, 1984
- Replaces texture map with summed-area texture map
  - $S(x,y)$ = sum of texels <= x,y
  - Need double range (e.g. 16 bit)
- Creation
  - Incremental sweep using previous computations
  - $S(x,y) = T(x,y) + S(x-1,y) + S(x,y-1) - S(x-1,y-1)$
- Accessing
  - $\Sigma\, T([x_1,x_2],[y_1,y_2]) = S(x_2,y_2) - S(x_1,y_2) - S(x_2,y_1) + S(x_1,y_1)$
  - Ave $T([x_1,x_2],[y_1,y_2])/((x_2 - x_1)(y_2 - y_1))$



Summed Area Tables

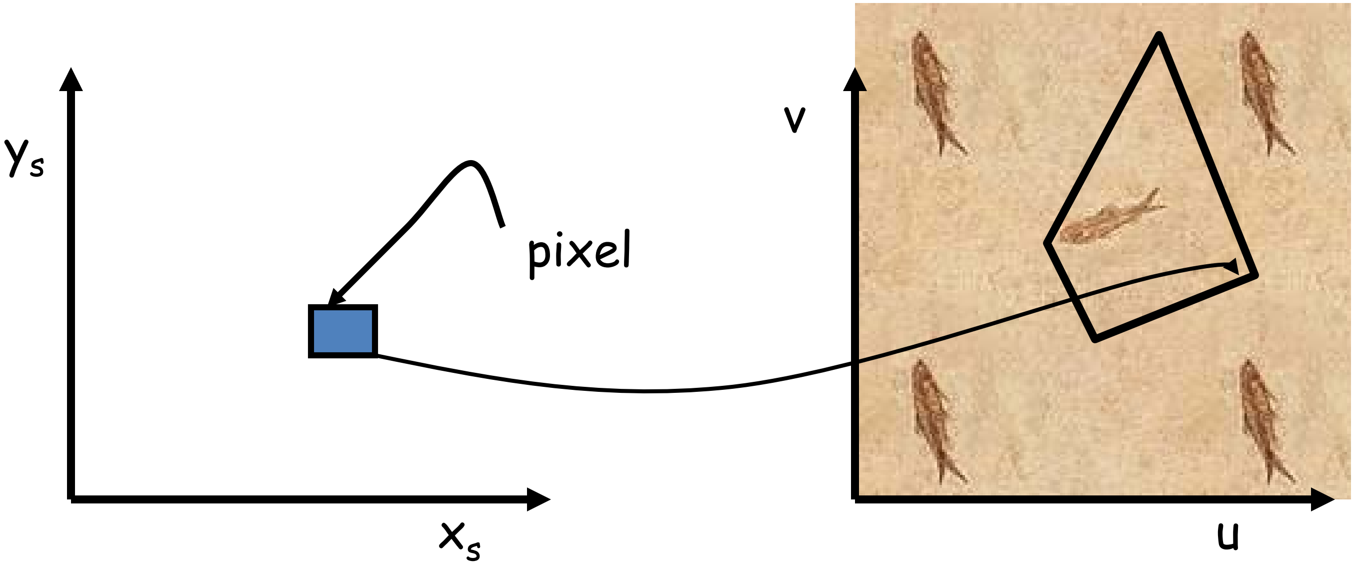
- A 2D table the size of the texture. At each entry (i,j), store the sum of all texels in the rectangle defined by (0,0) and (i,j).
- Given any axis aligned rectangle, the sum of all texels is easily obtained from the summed area table:

$$\mathbf{area} = \mathbf{A} - \mathbf{B} - \mathbf{C} + \mathbf{D}$$



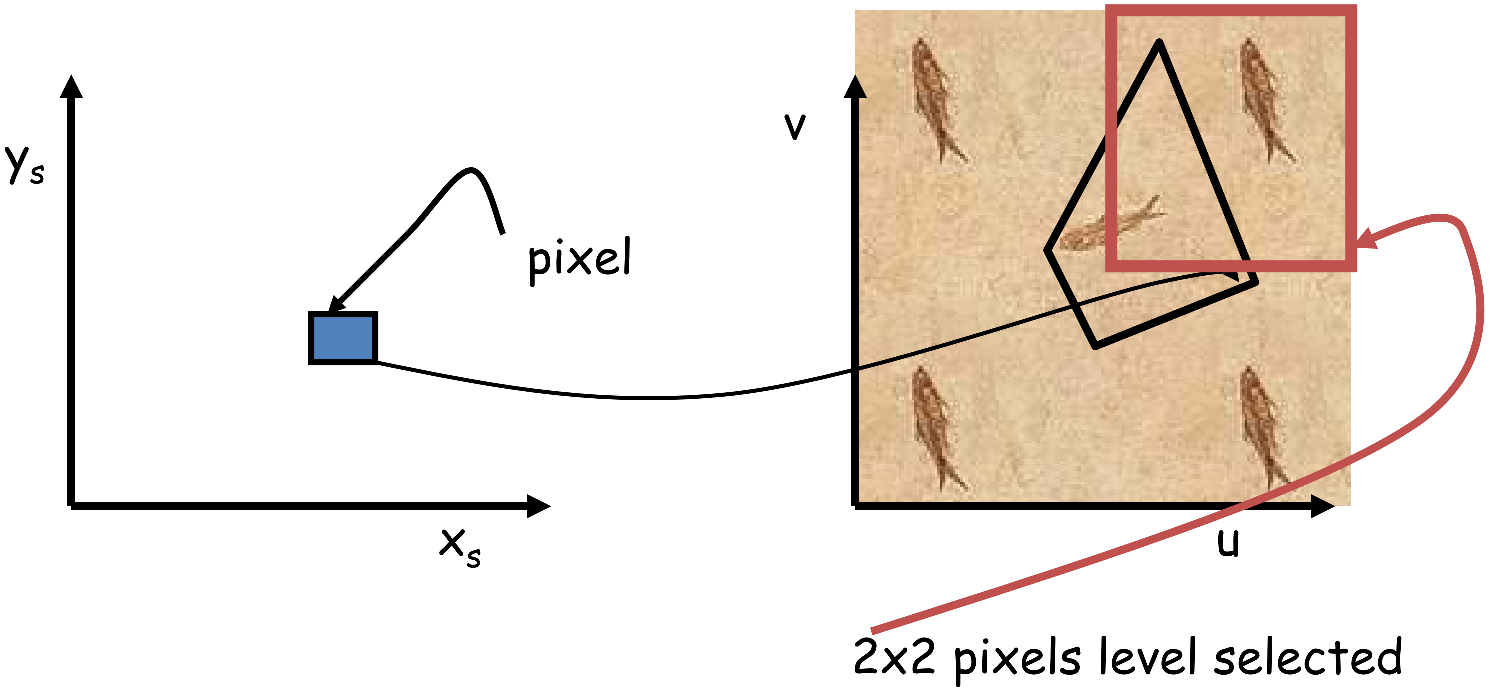
Quality considerations

- Pixel area maps to "weird" (warped) shape in texture space



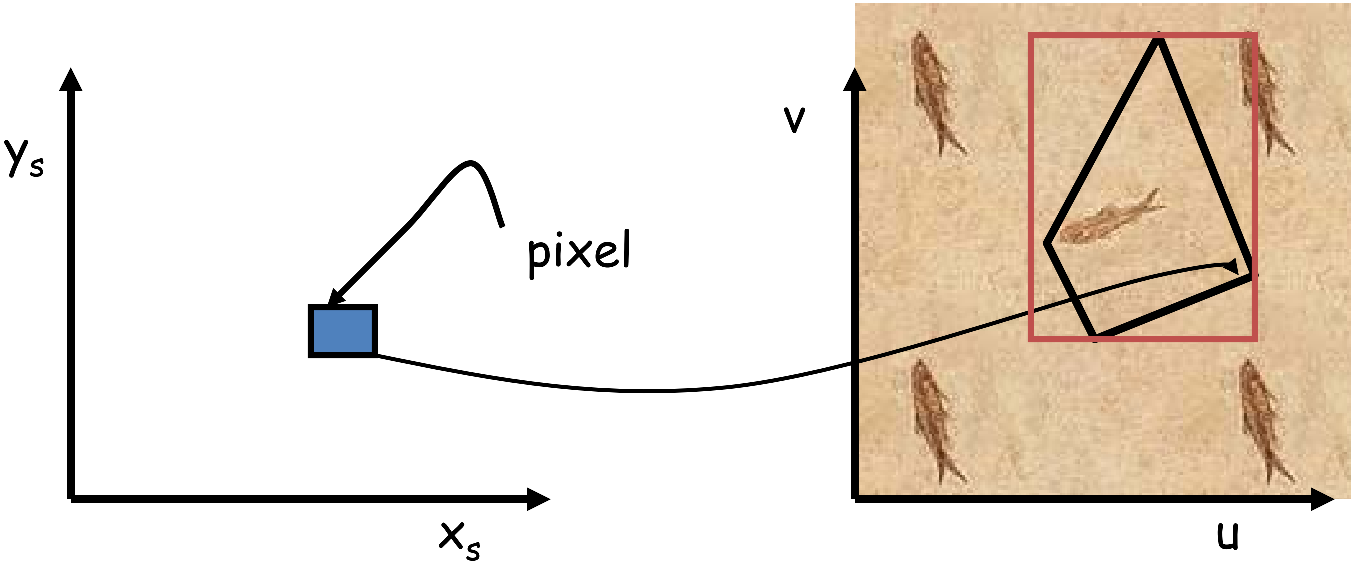
Mip-maps

- Find level of the mip-map where the area of each mip-map pixel is closest to the area of the mapped pixel.



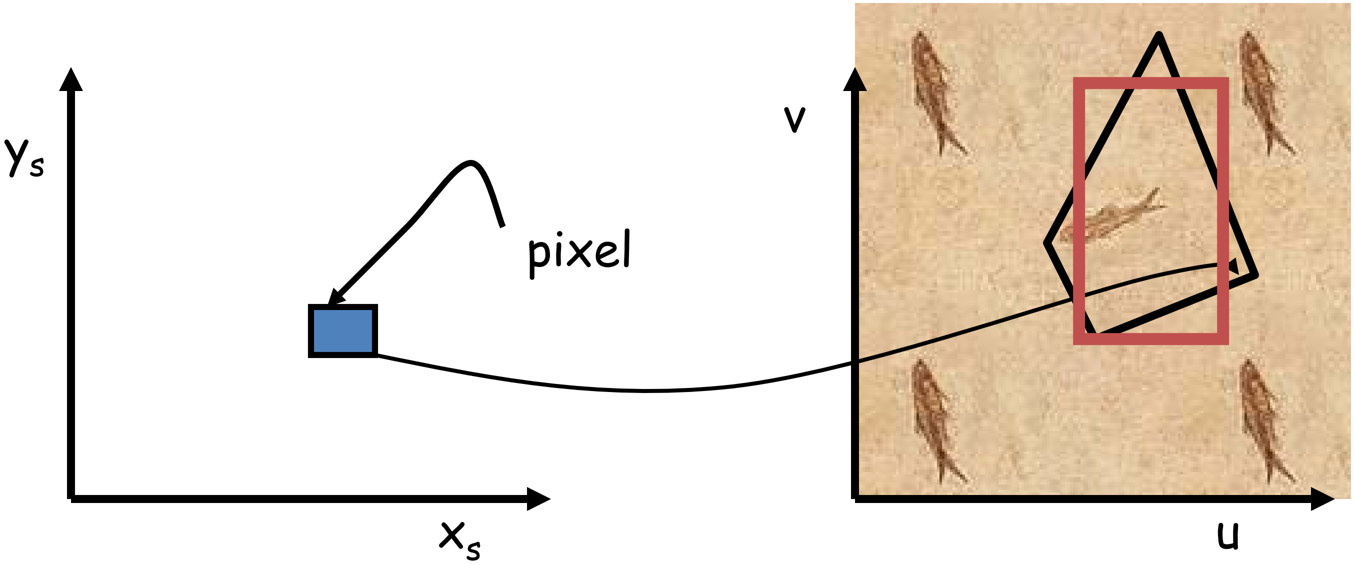
Summed Area Table (SAT)

- Determining the rectangle:
  - – Find bounding box and calculate its aspect ratio



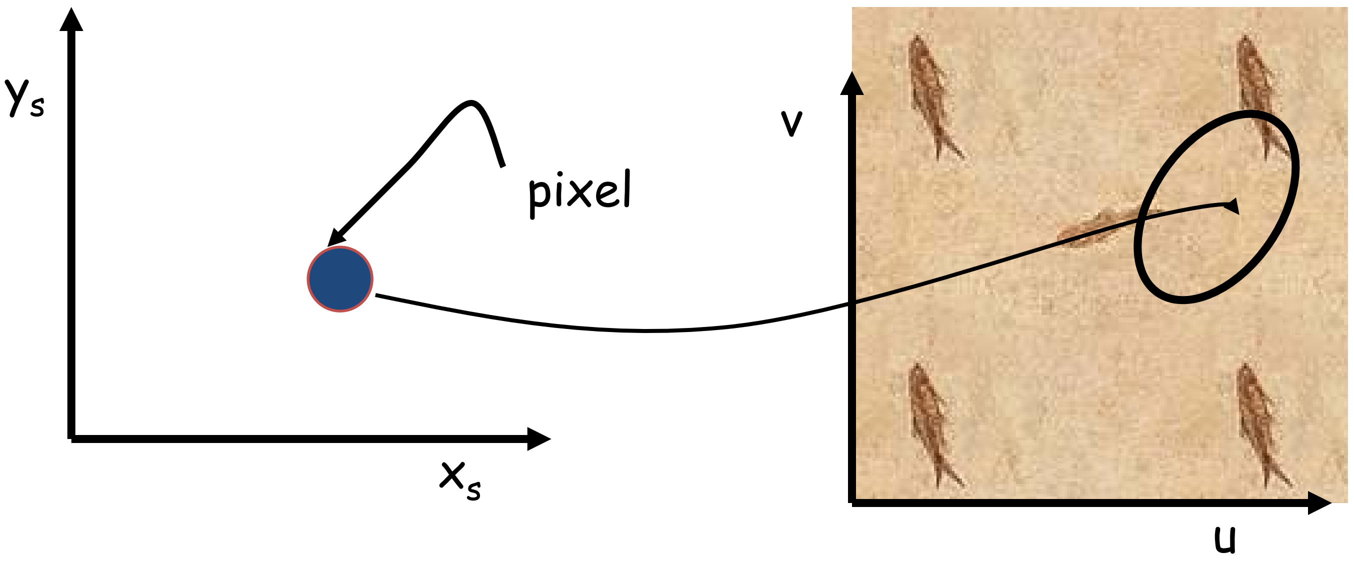
Summed Area Table (SAT)

- Determine the rectangle with the same aspect ratio as the bounding box and the same area as the pixel mapping.

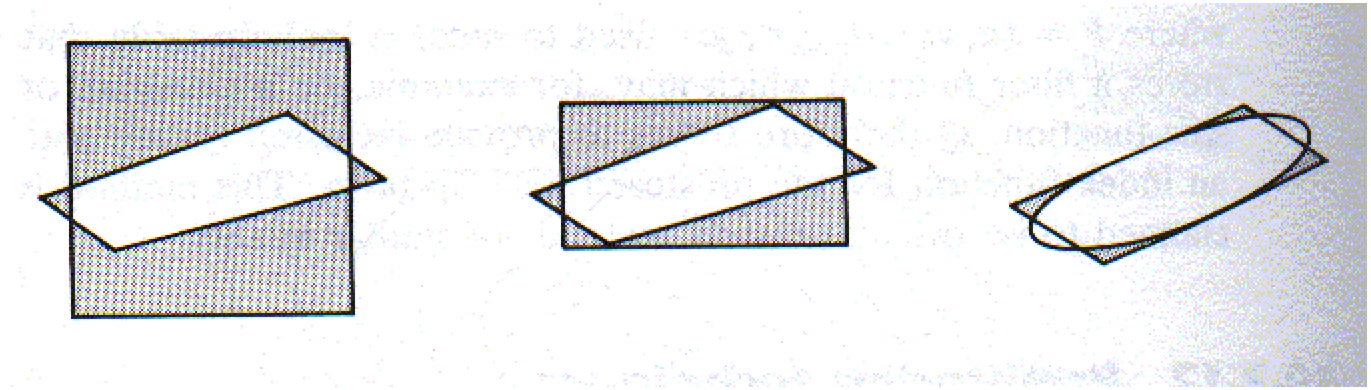


Elliptical Weighted Average (EWA) Filter

- Treat each pixel as circular, rather than square.
- Mapping of a circle is elliptical in texel space.



Texture Domain



Elliptical Weighted Average