Optical light bullets in a pure Kerr medium

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We show that small negative fourth-order dispersion can arrest spatiotemporal collapse of ultrashort pulses with anomalous dispersion in a planar waveguide with pure Kerr nonlinearity, resulting in (2 + 1)D optical bullets. Similarly to solitons, these bullets undergo elastic collisions. Since these bullets can self-trap from noisy Gaussian input beams and propagate without any power losses, this result may be used to realize experimentally stable, nondissipative optical bullets.

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The idea of an "optical bullet," a laser pulse localized in space and in time that propagates while retaining its spatiotemporal shape, has fascinated the nonlinear optics community ever since it was proposed by Silberberg,[1] and it remains one of the holy grails of nonlinear optics. To achieve an optical bullet requires a complete balance of the focusing Kerr nonlinearity with diffraction and anomalous dispersion. The corresponding mathematical model is either the two-dimensional or the three-dimensional cubic nonlinear Schrödinger equation (NLSE), depending on whether diffraction is limited to one transverse dimension (a planar waveguide) or two transverse dimensions (bulk medium), respectively. Since all solitary-wave solutions of the cubic NLSE in dimension $D \geq 2$ are unstable, until now it was believed that optical bullets are unstable in a pure Kerr medium. Therefore, in all previous theoretical and experimental studies, optical bullets were obtained through deviations from a pure Kerr nonlinearity. Thus, it was shown theoretically that optical bullets are stable in a saturable Kerr medium.[2] In Ref. 3, it was reported that spatiotemporal compression was achieved experimentally in a planar waveguide, but the stabilization was due to multiphoton absorption and Raman scattering, both of which are dissipative. Liu *et al.*[4] created an optical bullet in a planar waveguide with quadratic nonlinearity. In another experiment, stable optical bullets were realized in bulk media with normal time dispersion.[5] Since, however, in a pure Kerr medium pulses with normal dispersion do not undergo temporal compression, it was suggested that higher-order nonlinearities played a key role in the pulse compression and bullet stabilization. Recently, stable optical bullets were found in a medium with quadratic nonlinearity in the case of normal dispersion at the second harmonic.[6]

The standard model for propagation of solitons and optical bullets with anomalous dispersion is the dimensionless NLSE:

$$i\psi_z(z,\mathbf{x}) + \Delta\psi + |\psi|^{2\sigma}\psi = 0\,, \qquad \psi(0,\mathbf{x}) = \psi_0(\mathbf{x})\,, \quad (1)$$

where ψ is the electric field amplitude; z is the axial distance; $\mathbf{x} = (x_1, \ldots, x_D)$, where $(x_1, \ldots, x_{D-1})$ are the transverse spatial coordinates and x_D is time; $\Delta = \Delta_\perp + \partial x_D x_D$, where $\Delta_\perp = \partial_{x_1 x_1} + \ldots + \partial x_{D-1} x_{D-1}$ is diffraction and $\partial x_D x_D$ is anomalous dispersion; and $\sigma = 1$ for a Kerr medium. It is well known that the critical exponent of the NLSE is $\sigma^*_{\text{NLSE}} = 2/D$, i.e., when $\sigma < 2/D$, solutions of Eq. (1) remain bounded for all z and its solitary-wave solutions are stable, but when $\sigma \geq 2/D$, Eq. (1) admits of solutions that become singular (blowup, collapse) at a finite propagation distance z, and its solitary-wave solutions are unstable. Therefore, based on Eq. (1), it was concluded that in a pure Kerr medium, solitons (i.e., $D = 1$) are stable but optical bullets (i.e., $D = 2, 3$) are unstable.

In Eq. (1), physical time is no different than any of the spatial coordinates in the transverse plane. There is, however, a fundamental difference between space and time in spatiotemporal self-focusing, since the more comprehensive physical model includes higher-order dispersion terms but no such high-order diffraction terms. Therefore, a more accurate model of spatiotemporal self-focusing in a Kerr medium is given by the NLSE with anisotropic high-order dispersion, where by anisotropy we mean that the high-order dispersion terms are not isotropic in $\mathbf{x}$. In a recent study[7] it was shown that the critical exponent for the NLSE with a negative one-dimensional fourth-order dispersion (4OD), i.e.,

$$i\psi_z(z,\mathbf{x}) + \Delta\psi - \alpha^2\psi_{x_D x_D x_D x_D} + |\psi|^{2\sigma}\psi = 0\,, \qquad (2)$$

where $\alpha^2 > 0$, is given by $\sigma^*_{\text{anisotropic}}(D) = 2/(D - 1/2)$. Therefore, when $\sigma < \sigma^*_{\text{anisotropic}}$ solutions of Eq. (2) do not collapse, and its waveguide solutions are stable. As we shall see, these new critical exponents imply that collapse of ultrashort laser pulses can be arrested by negative 4OD in a planar waveguide but not in a bulk medium. Indeed, we show that one may be able to utilize 4OD to create a stable, nondissipative optical bullet in a planar waveguide geometry. To the best of our knowledge this constitutes the first theoretical demonstration of stable, nondissipative (1 + 2)D optical bullets in a pure Kerr medium.

The propagation of a linearly polarized laser pulse in a bulk Kerr medium can be modeled by

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$$A_{zz}(z,x,y,t) + 2ik_0 A_z + \Delta_\perp A + \sum_{m=1}^{\infty} \frac{1}{m!} \frac{\partial^m (k^2)}{\partial \omega^m} (i\partial_t)^m A + \frac{4n_2 n_0 \omega_0^2}{c^2} |A|^2 A = 0\,,$$

where A is the slowly varying amplitude, $\Delta_\perp = \partial_{xx} + \partial_{yy}$ is diffraction, $k^2(\omega) = \omega^2 n_0^2(\omega)/c^2$, and $k_0 = k(\omega_0)$. Let us change to the moving-frame coordinate system $\tilde{x} = x/r_0$, $\tilde{y} = y/r_0$, $\tilde{z} = z/2L_{\text{diff}}$, $\tilde{t} = (t - z/c_g)/T$, and $\psi = r_0 k_0 \sqrt{4n_2/n_0}\, A$, where r_0 is the initial pulse width, $L_{\text{diff}} = r_0^2 k_0$ is the diffraction length, T is the pulse duration, and $c_g = 1/k'(\omega_0)$ is the group velocity. When the pulse is sufficiently short that $T \ll L_{\text{diff}} \sqrt{|k_0 k_{\omega\omega}|}$, the equation for the nondimensional envelope ψ can be approximated with[8]

$$i\psi_z(z,x,y,t) + \Delta_\perp \psi + |\psi|^2 \psi - \gamma_2 \psi_{tt} - i\gamma_3 \psi_{ttt} + \gamma_4 \psi_{tttt} = 0\,, \quad (3)$$

where the tildes have been dropped, $\gamma_2 = L_{\text{diff}}/L_{\text{ds}}$, where $L_{\text{ds}} = T^2/k''(\omega_0)$ is the dispersion length, $\gamma_3 = r_0^2 k_{\omega_0} k'''_{\omega_0}/(3T^3)$ is third-order dispersion (3OD), $\gamma_4 = r_0^2 k_{\omega_0} k''''_{\omega_0}/(12T^4)$ is 4OD, and higher-order dispersion terms are neglected. Similarly, the propagation in a planar waveguide is given by

$$i\psi_z(z,x,t) + \psi_{xx} + |\psi|^2 \psi - \gamma_2 \psi_{tt} - i\gamma_3 \psi_{ttt} + \gamma_4 \psi_{tttt} = 0\,. \quad (4)$$

Let us first consider the case of a laser pulse that operates in the anomalous time-dispersion regime (i.e., $\gamma_2 < 0$) at wavelengths for which the effect of 3OD is minimized,[9] as, e.g., in the case of dispersion-flattened fibers. In this case, pulse propagation in a planar waveguide geometry is governed by

$$i\psi_z(z,x,t) + \psi_{xx} + |\psi|^2 \psi - \gamma_2 \psi_{tt} + \gamma_4 \psi_{tttt} = 0\,. \quad (5)$$

When $\gamma_2 < 0$ and $\gamma_4 < 0$, Eq. (5) is (after the rescaling $x_D = t/\sqrt{-\gamma_2}$) a special case of Eq. (2) with $\sigma = 1$ and $D = 2$. Since $\sigma^*_{\text{anisotropic}}(2) = 4/3$, we conclude that in a planar waveguide geometry small negative 4OD always arrests spatiotemporal collapse. In addition, since $\sigma < \sigma^*$, the solitary-wave solutions of Eq. (5) are stable.[7] To confirm this theoretical prediction, in Fig. 1 we solve Eq. (5) with the Gaussian input beam $\psi_0 = 2\sqrt{1.25 N_c}\exp(-x^2 - t^2)$, where $N_c \approx 1.86$ is the dimensionless critical power. In the absence of 4OD the solution undergoes collapse. As predicted, collapse is arrested by negative 4OD, and subsequently the pulse undergoes stable focusing–defocusing oscillations that can be interpreted as an optical bullet (Fig. 2). The robustness of bullet formation is manifested by repeating this simulation with the same input beam, to which we add input focusing and chirping as well as 5% complex-valued random noise both in x and in t; i.e., $\psi_0 = 2\sqrt{1.25 N_c}\exp(-x^2 - t^2)\exp[-i/4(x^2 + t^2)][1 + 0.05\,\text{rand}(x,t)]$. Moreover, these bullets are also robust when they collide (Fig. 3).

Negative 4OD is unable to arrest collapse in a bulk medium. Indeed, in that case the propagation is given by

$$i\psi_z(t,x,y,z) + \psi_{xx} + \psi_{yy} + |\psi|^2 \psi - \gamma_2 \psi_{tt} + \gamma_4 \psi_{tttt} = 0\,. \quad (6)$$

When $\gamma_2 < 0$ and $\gamma_4 < 0$, Eq. (6) is a special case of Eq. (2) with $\sigma = 1$ and $D = 3$. Since $\sigma^*_{\text{anisotropic}}(3) = 4/5$, we conclude that in a bulk medium negative 4OD does not arrest collapse. To illustrate this we show in Fig. 4 the solution of Eq. (6) with the Gaussian input beam $\psi_0 = 4\exp(-x^2 - y^2 - t^2)$. Indeed, collapse is only delayed, but not arrested, by negative 4OD.

Although there is no rigorous theory for the case of a positive 4OD, the simulations of Figs. 1 and 4 suggest that positive 4OD arrests collapse in both the planar waveguide and the bulk media case through strong temporal dispersion. As a result, in the positive 4OD case there is only a single focusing event followed by complete defocusing. Hence, one cannot stabilize optical bullets by use of positive 4OD.

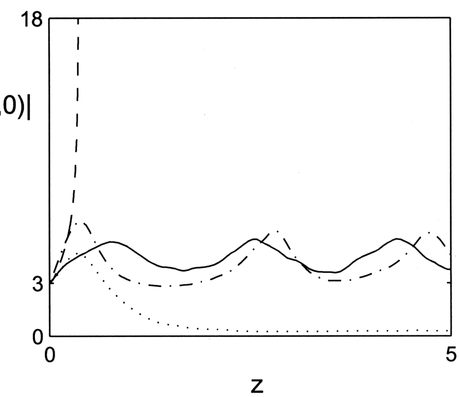

Fig. 1. Amplitude of the solution of Eq. (5) with $\gamma_2 = -1$, and $\gamma_4 = 0$ (dashed curve), $\gamma_4 = -0.04$ (solid curve), $\gamma_4 = -0.04$ with focusing, chirping, and input noise (dashed–dotted curve), and $\gamma_4 = 0.04$ (dotted curve).

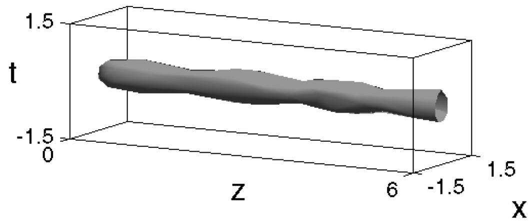

Fig. 2. Isointensity plot of the solid curve solution from Fig. 1.

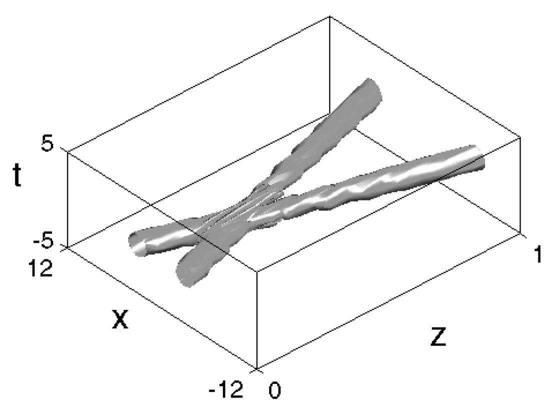

Fig. 3. Elastic collision of two bullets.

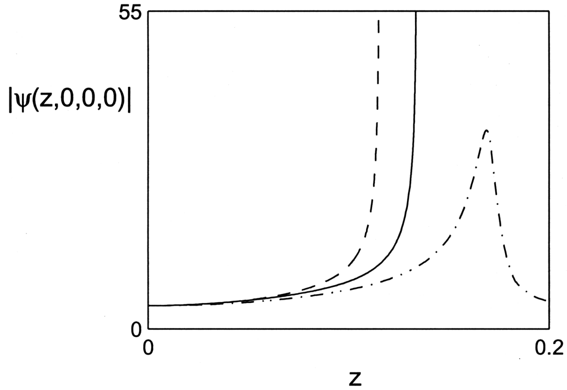

Fig. 4. Amplitude of the solution of Eq. (6) with $\gamma_2 = -1$ and $\gamma_4 = 0$ (dashed curve), $\gamma_4 = -0.05$ (solid curve), and $\gamma_4 = 0.05$ (dashed–dotted curve).

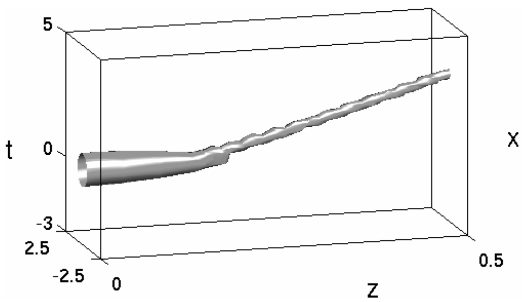

Fig. 5. Optical bullet at 1.8 μm.

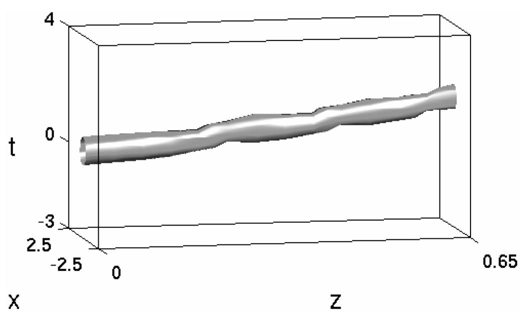

Fig. 6. Same as Fig. 5 with the addition of a shock term.

We now study the effect of 3OD on optical bullets. Let us first note that, unlike γ_2 and γ_4, the sign of γ_3 has no effect on the dynamics. Indeed, if we change γ_3 to $-\gamma_3$ and t to $-t$, Eq. (3) remains unchanged. Therefore, the sign of γ_3 determines only the direction of the effect of 3OD along the t axis. Indeed, when $\gamma_4 = 0$, solutions of Eq. (4) satisfy

$$\frac{\mathrm{d}}{\mathrm{d}z}\int t|\psi|^2\mathrm{d}x\mathrm{d}t = 2\gamma_2\,\mathrm{Im}\int \psi_0\psi_{0,t}^*\mathrm{d}x\mathrm{d}t + 3\gamma_3\int |\psi_t|^2\mathrm{d}x\mathrm{d}t\,. \quad (7)$$

Therefore, as a result of 3OD, the center of mass moves in the positive (negative) t direction when $\gamma_3 > 0$ ($\gamma_3 < 0$).

To test whether optical bullets can be realized experimentally, we wanted to solve Eq. (4) with physical parameters. For example, for a bullet propagating through a silica planar waveguide with $r_0 = 37.7$ μm, $T = 30$ fs, and $\lambda = 1.8$ μm, the Sellmeier equation for silica[10] gives $\gamma_2 = -0.5$, $\gamma_3 = 0.025$, and $\gamma_4 = -9 \times 10^{-4}$. Because of limitations of our computer resources, we could not solve Eq. (4) reliably using these values. Therefore, in the simulation shown in Fig. 5 we solve Eq. (3) by use of $\gamma_2 = -1$, $\gamma_3 = 0.052$, and $\gamma_4 = -0.0014$ for the input pulse $\psi_0 = 2\sqrt{2N_c}\exp(-x^2 - t^2)$. These parameters were chosen to mimic the physical ones, i.e., $-\gamma_2 \gg \gamma_3 \gg -\gamma_4$. Although 3OD is considerably larger in magnitude than 4OD, the pulse still undergoes spatiotemporal compression that results in an optical bullet. In fact, the only noticeable effect of 3OD is that the bullet moves in the positive t direction, in accordance with Eq. (7). We note that the peak intensity in Fig. 5 is $\approx 4 \times 10^{16}$ W/m^2, which is well below the threshold intensity for optical damage in fused silica.[11] Finally, we repeated this simulation with the addition of the shock term $0.035i(|\psi|^2\psi)_t$ to Eq. (4) and confirmed that it did not disrupt the bullet behavior (Fig. 6).

In summary, we have used an abstract result on critical exponents of anisotropic NLSEs, as well as numerical simulations, to show that with a proper choice of wavelength (i.e., wavelength that corresponds to anomalous dispersion and negative 4OD) it may be possible to realize optical bullets in planar waveguides with a pure Kerr nonlinearity. This theoretical observation was recently suggested as a possible explanation for the experimental results of Cheskis *et al.*[12]

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References

1. Y. Silberberg, Opt. Lett. **15,** 1282 (1990).
2. K. J. Blow, N. J. Doran, and D. Wood, IEEE J. Quantum Electron. **27,** 2060 (1991).
3. H. S. Eisenberg, R. Morandotti, Y. Silberberg, S. Bar-Ad, D. Ross, and J. S. Aitchison, Phys. Rev. Lett. **87,** 043902 (2001).
4. X. Liu, L. J. Qian, and F. W. Wise, Phys. Rev. Lett. **82,** 4631 (1999).
5. I. G. Koprinkov, A. Suda, P. Q. Wang, and K. Midorikawa, Phys. Rev. Lett. **84,** 3847 (2000).
6. I. N. Towers, B. A. Malomed, and F. W. Wise, Phys. Rev. Lett. **90,** 123902 (2003).
7. G. Fibich, B. Ilan, and S. Schochet, Nonlinearity **16,** 1809 (2003).
8. G. Fibich and G. C. Papanicolaou, Opt. Lett. **22,** 1379 (1997).
9. A. Höök and M. Karlsson, Opt. Lett. **18,** 1388 (1993).
10. I. H. Malitson, J. Opt. Soc. Am. **55,** 1205 (1965).
11. C. B. Schaffer, A. Brodeur, and E. Mazur, Meas. Sci. Technol. **12,** 1784 (2001).
12. D. Cheskis, S. Bar-Ad, R. Morandotti, J. S. Aitchison, H. S. Eisenberg, Y. Silberberg, and D. Ross, Phys. Rev. Lett. **91,** 223901 (2003).