The Dynamics of Price Elasticity of Demand in the Presence of Reference Price Effects

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The authors derive an expression for the price elasticity of demand in the presence of reference price effects that includes a component resulting from the presence of gains and losses in consumer evaluations. The effect of reference price is most noticeable immediately after a price change, before consumers have had time to adjust their reference price. As a result, immediate-term price elasticity is higher than long-term elasticity, which describes the response of demand long after a price change, when reference price effects are negligible. Furthermore, because of the differential effect of gains and losses, immediate-term price elasticity for price increases and price decreases is not equal. The authors provide a quantitative definition for the terms immediate term *and* long term*, using the average interpurchase time and the discrete "memory" parameter. Practical consequences of the distinction between immediate- and long-term elasticities for the estimation and use of elasticity values are discussed.*

Keywords: *Reference price; price elasticity; immediate term; promotional elasticity*

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1. INTRODUCTION

Price elasticity of demand is the percentage change in quantity demanded as a result of a 1 percent change in price. It is defined as

$$\varepsilon = \frac{\frac{dQ}{Q}}{\frac{dp}{p}} = \frac{dQ}{dp}\frac{p}{Q}, \tag{1}$$

where p is price and $Q(p)$ is market demand. Numerous factors can affect the price elasticity of demand, including closeness of substitute products, importance of the good in terms of expenditure, time for adjustment, product durability, and range of uses. In this study, we explore the effect of reference price on the price elasticity of demand, an effect that has not been considered previously. Specifically, we examine the dynamics of price elasticity that result from changes in quantity demanded over time. Under this framework, changes in demand occur once there is a price change, and deviations between this new price and consumers' reference price occur. Consumers' reference price adjustments, a process that evolves over time, yield changes in these price deviations. The impact of these changes is reflected in the time dependence of quantity demanded that creates the dynamics of price elasticity.

Changes in price elasticity of demand over time have been investigated before in the marketing literature. Such studies, however, were mainly concerned with the changes of price elasticity along the product life cycle (Mickwitz 1959; Simon 1979; Liu and Hanssens 1981; Lilien and Yoon 1988).

Research in the area of reference price effects on consumer behavior has investigated various aspects of these effects. One stream of research is aimed at exploring reference price effects on demand. In this context, the main objective is deriving optimal pricing strategies in the presence of reference price effects. In a recent study, Fibich, Gavious and Lowengart (2003) studied retailers' pricing strategies in the presence of asymmetric effects of reference price over a finite or infinite planning horizon. Their findings indicate that the optimal pricing strategy would be a penetration or skimming at the introductory stage and a constant price afterward. Observing current retailers' pricing practices in the marketplace, it can be seen that the high-low pricing strategy is being used quite often. Thus, it would be very useful to examine the dynamics of price elasticity. The findings of the present study reveal large differences between the elasticities of a price increase and a price decrease and the time dependence of these results. Furthermore, our results suggest that the reference price can have a considerable effect on price elasticity. For example, by applying our analysis to empirical data, we show that during the first few weeks after a price change, reference price effects on the price elasticity of demand for peanut butter are higher than price effects.

Reference price can be defined as the price consumers have in mind and to which they compare the shelf price of a specific product (Winer 1986, 1989; Mayhew and Winer 1992; Kalwani, Rinne, and Sugita 1990). Differences between the reference price r and the shelf price p affect the demand for that brand: when $r > p$, consumers are likely to sense a *gain* that will increase demand for this brand, and when $r < p$, consumers are likely to sense a *loss* that will negatively affect demand. Consumers construct their internal reference price through their shopping experiences with the product over time. Consequently, when a retailer changes its price, there is a time lag until consumers adjust their reference price to the new price. During this time lag, the existence of gain or loss evaluations affects demand for the product. With time, however, these effects decrease, and quantity demanded is less affected by reference price evaluations. As a result, the price elasticity of demand depends not only on price but also on whether we are interested in the demand level immediately following a price change or much later in time.

We start our analysis with a discrete model of reference price formation, which we use to derive expressions for immediate- and long-term price elasticities. Immediate-term price elasticity corresponds to the change in demand right after the price change occurred, when reference price effects are maximal. Long-term price elasticity corresponds to the change in demand long after the price change occurred, when reference price effects have become negligible. Long-term elasticity is, therefore, equal to regular price elasticity (i.e., elasticity in the absence of the reference price). We show that immediate-term price elasticity is always greater than regular price elasticity (Section 4). In fact, we explicitly calculate the relative change in immediate-term elasticity (RCIE) and show that it is equal to the relative effect of the reference price on demand, compared with that of the actual price (equation (9)).

There is no explicit time scale in the discrete model that can be used to quantify the terms *immediate term* and *long term*. We can get this quantification, however, by using a continuous model for reference price formation. In addition, with this approach, we derive a general expression for the price elasticity of demand with reference price effects (equation (17)), which depends both on price and length of time elapsed since the price change, such that the expressions derived under the discrete approach for immediate- and long-term elasticities correspond to its two extreme cases (Section 6.2).

The theoretical distinction between short- and long-term price elasticities of demand has practical consequences for the estimation of elasticity values. In Section 9, we show how to determine efficiently these elasticities by using a specific sampling scheme based on separating the sampling data of quantity demanded into two sets: the first consisting of sampling shortly after a price change and the second consisting of sampling long after the price change. With this approach, price and reference price effects are separated, allowing one to determine price elasticity more accurately (i.e., not mixing immediate- and long-term elasticities) and with less effort (i.e., fewer sampling points).

We extend our analysis to competitive situations, in which demand for product A is also dependent on the price of the other products in the category and vice versa. The results show that reference price effects on price elasticity are the same as in a monopoly case. As reference price effects constitute the transactional utility component of promotional elasticity (Blattberg and Neslin 1989), immediate-term elasticity equals promotional elasticity in the case of a monopoly (Section 12). Even in competitive situations, in which brand switching accounts for most of the increase in promotional elasticity, transactional utility still exists and has the same impact on promotional elasticity as in the monopoly situation. Other extensions to the basic model include multiple price changes (Section 11) and consumers' heterogeneity (Section 13). In all cases, we show that there is no difference in the qualitative results of the general model and the extended cases.

To depict the relevance of our theoretical development to real-life marketing situations, we provide an empirical illustration of determining immediate- and long-term elasticities (Section 5). We also show how to determine the time scale parameter T_{RP}, which distinguishes between immediate- and long-term time durations (Section 7).

In sum, the lack of research that examines the price elasticity of demand in the presence of the reference price, in general, and its asymmetric effects, in particular, is the main drive for this research. Furthermore, since reference price formation modeling approaches are based on consumers' memory-decaying process of past prices, there is a clear need to identify the time scale for this process and explore the dynamics of price elasticity. Such identification will enable modelers to better understand reference price effects on consumer behavior over time.

This article, therefore, addresses these two main issues. We derive an expression for the price elasticity of demand in the presence of reference price effects that have not been identified before. This identification can enable marketers to better estimate price elasticity for their products by understanding the asymmetric nature and the contribution of gains (losses) to this estimation. Furthermore, we propose an efficient method to sample data to get price elasticity estimations. Noting that reference price effects are time dependent, we derive a characterization of these effects over time. This formulation enables us to make the distinction between the short- and long-term effects of reference price on demand. To show the applicability and relative ease of use of our analytical approach, we use published data to illustrate the calculation of price elasticities.

2. MODELING REFERENCE PRICE

In this section, we give a short review of discrete reference price modeling (for more details, see Greenleaf 1995; Kopalle, Rao, and Assunção 1996).

The process of establishing an internal reference price is constructed by consumers through personal experiences such as purchasing, observing, or being exposed to intentional and unintentional price information. Since previous exposures have decaying weights in consumer evaluation, this process can be modeled by (e.g., Kalyanaram and Little 1994; Lattin and Bucklin 1989)

$$r_n = \eta r_{n-1} + (1-\eta) p_{n-1}, \tag{2}$$

where p_n and r_n are shelf and reference prices at the nth buy, respectively, and η is a discrete "memory" parameter that depends on the product category.

To model reference price effects on consumer purchasing quantity, we use the common model in the literature for aggregate reference price effects (e.g., Kopalle and Winer 1996; Kopalle et al. 1996) and denote market demand in the absence of reference price effects by $Q_{no\text{-}ref}(p)$.

In the presence of reference price effects, the demand function[1] is given by (e.g., Greenleaf 1995)

$$Q = Q_{no\text{-}ref}(p) - \gamma(p - r). \tag{3}$$

The effects of losses and gains on consumer demand are separated by using the *asymmetric* model,

$$\gamma = \begin{cases} \gamma_{gain} & p \le r \\ \gamma_{loss} & p > r \end{cases}, \tag{4}$$

where γ_{gain} and γ_{loss} are positive constants.

3. PRICE ELASTICITY IN THE PRESENCE OF REFERENCE PRICE EFFECTS (DISCRETE APPROACH)

To simplify the presentation, we begin with the case of a monopoly and a single price change,

$$p_j = \begin{cases} p_{old} & j < n \\ p_{new} & j \ge n \end{cases}.$$

Our results can be extended to a competitive situation (Section 10) and for multiple price changes (Section 11).

To calculate the price elasticity of demand in the presence of reference price effects, we note that from (2) and (3), it follows that

$$\begin{aligned} Q_n &= Q_{no-ref}(p_n) - \gamma p_n + \\ &\gamma(1-\eta)[p_{n-1} + \eta p_{n-2} + \eta^2 p_{n-3} + \cdots]. \end{aligned} \tag{5}$$

In the absence of reference price effects ($\gamma = 0$), the price elasticity of demand is given by

$$\varepsilon_{no-ref}(p) = Q'_{no-ref}(p) \frac{p}{Q_{no-ref}(p)}, \tag{6}$$

where

$$Q'_{no-ref}(p) = \frac{d}{dp} Q_{no-ref}(p).$$

When retailers plan a short-term price change (e.g., price promotion), the relevant elasticity information is the *immediate* change in demand following a price change.[2] In light of equation (5), this immediate change is given by

$$\frac{\partial Q_n}{\partial p_n} = Q'_{no-ref}(p_n) - \gamma.$$

Therefore, the general expression for *immediate-term* price elasticity is given by

$$\varepsilon_{immediate-term}(p) = \left[Q'_{no-ref}(p) - \gamma\right] \frac{p}{Q_{no-ref}(p) - \gamma(p-r)}. \qquad (7)$$

Let us consider first a case in which previous price changes have occurred sufficiently long ago, so that just before the current price change, $p \approx r$. Thus, $p_{old} = r$ (but not $p_{new} = r$). Just after the price change, however, the difference between the new price p_{new} and r is maximal, corresponding to the case of immediate-term elasticity,[3] thus resulting in

$$\varepsilon_{immediate-term}(p) = [Q'_{no-ref}(p) - \gamma] \frac{P}{Q_{no-ref}(p)}. \qquad (8)$$

In contrast, when retailers plan on maintaining the price change constant for a long time period, a more relevant parameter is the *long-term* effect of the price change on demand (i.e., *long-term* price elasticity). In this case, consumers are exposed to the new price level long enough so that there is a minimal gap between their reference price and the new price, namely, $p \approx r$. Consequently, reference price effects are minimal, and $Q \approx Q_{no\text{-}ref}$. Therefore, *long-term* price elasticity is equal to price elasticity in the absence of reference price effects:

$$\varepsilon_{long-term} = \varepsilon_{no-ref}.$$

4. IMMEDIATE-TERM PRICE ELASTICITY OF DEMAND

From now on, we focus on immediate-term price elasticity, in which reference price effects are most important. The importance of reference price effects in the price elasticity of demand can be characterized by the relative change in immediate-term price elasticity (RCIE) due to the reference price effect:

$$RCIE = \frac{\varepsilon_{immediate-term} - \varepsilon_{no-ref}}{\varepsilon_{no-ref}}.$$

Using (6) and (8), we have

$$RCIE = \frac{\gamma}{-Q'_{no-ref}(p)}. \qquad (9)$$

From relation (9), we can draw the following conclusions:

1. As demand decreases monotonically with price—that is, $Q'_{no-ref}(p) < 0$—immediate-term price elasticity is always greater than regular price elasticity, namely, $RCIE > 0$.
2. The relative change in immediate-term price elasticity is equal to the relative effect of the reference price $(p - r)$ on demand, compared with that of the actual price p in the absence of reference price effects:

$$RCIE = \frac{\textit{effect of } (p-r) \textit{ on demand}}{\textit{effect of } p \textit{ on demand}}.$$

3. As reference price effects are asymmetric (equation 4), the relative increase in immediate-term price elasticity for a price increase and decrease is not equal:

$$RCIE = \begin{cases} \dfrac{\gamma_{loss}}{-Q'_{no-ref}(p)} & \text{for a price increase,} \\ \dfrac{\gamma_{gain}}{-Q'_{no-ref}(p)} & \text{for a price decrease.} \end{cases}$$

This observation is consistent with the large body of empirical evidence that the price elasticity of demand for a price increase and a price decrease is different.

4.1. Linear Demand Function

For completeness, we present the results for the case in which the demand function decreases linearly in p,

$$Q_{no\text{-}ref}(p) = a - \delta p, \qquad a,\ \delta > 0. \qquad (10)$$

The immediate- and long-term price elasticities of demand are given by

$$\varepsilon_{immediate-term} = \frac{-\delta - \gamma}{a - \delta p} p, \quad \varepsilon_{long-term} = \frac{-\delta}{a - \delta p} p = \varepsilon_{no-ref}.$$

As already observed, the relative change in immediate-term elasticity is equal to the relative effect of the reference price on demand, compared with that of the actual price:

$$RCIE = \frac{\gamma}{\delta}.$$

Note that in the case of a linear demand function, *RCIE* does not depend on p.

5. EMPIRICAL ILLUSTRATION: DISCRETE FORMULATION

In this section, we show how our theoretical results can be applied to a real-life situation. We use the empirical data for peanut butter (Greenleaf 1995). In this study, the demand function in the presence of reference price effects was estimated as

$$Q_{no\text{-}ref} = 308.3 - 1{,}878.9p,\ \gamma_{loss} = 2{,}088.4,\ \gamma_{gain} = 11{,}330, \qquad (11)$$

where p is measured in dollar/ounces, and Q is measured in ounces. The average price is $p_{av} = \$2.57/(28\text{-ounce jar})$.[4]

We estimate $\varepsilon_{immediate\text{-}term}$ for a price increase using equation (8) with $\gamma = \gamma_{loss}$:

$$\varepsilon^{loss}_{immediate-term}(p) = \frac{1{,}878.9 + 2{,}088.4}{308.3 - 1{,}878.9p}\,p.$$

Thus, at $p = p_{av}$,

$$\varepsilon^{loss}_{immediate-term}(p_{av}) = 2.68.$$

Similarly, we can estimate $\varepsilon_{immediate\text{-}term}$ for a price decrease using equation (8) with $\gamma = \gamma_{gain}$:

$$\varepsilon^{gain}_{immediate-term}(p) = \frac{1{,}878.9 + 1{,}1330}{308.3 - 1{,}878.9p}\,p.$$

Therefore,

$$\varepsilon^{gain}_{immediate-term}(p_{av}) = 8.93.$$

Finally, we estimate $\varepsilon_{no\text{-}ref}$ using equation (6):

$$\varepsilon_{no-ref}(p) = \frac{1{,}878.9}{308.3 - 1{,}878.9p}\,p.$$

Therefore,[5]

$$\varepsilon_{no\text{-}ref}(p_{av}) = 1.27.$$

We thus see that

$$RCIE = \begin{cases} \dfrac{2{,}088.4}{1{,}878.9} = 111\% & \text{for a price increase,} \\ \dfrac{1{,}1330}{1{,}878.9} = 603\% & \text{for a price decrease.} \end{cases}.$$

This empirical illustration shows that the effect of reference price on immediate-term price elasticity can be quite large. In fact, immediately after a price change, reference price effects are larger than price effects, both for a price decrease and even more so for a price increase. Another implication is the large difference between immediate-term price elasticity for the price increase and decrease, which comes from the differential effect of losses and gains.

Our analytical approach yields general results and can accommodate any relation between gains and losses. In particular, our model does not determine whether consumers are loss aversive. Furthermore, it does not determine whether elasticity is larger for price increases or decreases. These issues are determined by the empirical data that serve as an input to our model, namely, the values of γ_{loss} and γ_{gain}. Thus, the results in our empirical illustration merely reflect the relation between loss and gain effects that was found in Greenleaf (1995), which, in this case, was such that $\gamma_{loss} < \gamma_{gain}$. One explanation for the counterintuitive relation between gains and losses at the aggregate-level analysis can be attributed to the idea that price promotions might have a larger effect on demand for the low-usage rate and high-price sensitivity consumer segment than for the less-price-sensitive segment. As a result, a larger impact will be observed for gains than losses at the aggregate-level analysis, even though each household will exhibit a greater effect for losses than gains (Greenleaf 1995).

It should be noted that the analytical elasticity formulations that were obtained in this study are not dependent on the nature of the data that are used for calculating elasticities. The stability of the prices, for example, would not have an effect on these calculations; rather, it would be reflected in the time duration between price changes, as reflected in equation (23).

6. CONTINUOUS FORMULATION

The expressions for immediate- and long-term price elasticities correspond to the two extreme cases of changes in demand: immediately after a price change, when reference price effects are maximal, and long after the price change, when reference price effects have disappeared. There is, however, no time scale in these expressions that would allow us to know, for example, whether the change in demand 2 months after a price change is characterized by immediate-term or long-term price elasticities. To answer this question, we derive an expression for price elasticity using a continuous-time model for a reference price formation. This expression depends continuously on the time that elapsed from the price change, such that the expressions for immediate- and long-term price elasticities correspond to its two limiting cases. As a result, we can quantify the terms *immediate-term* and *long-term* price elasticity of demand.

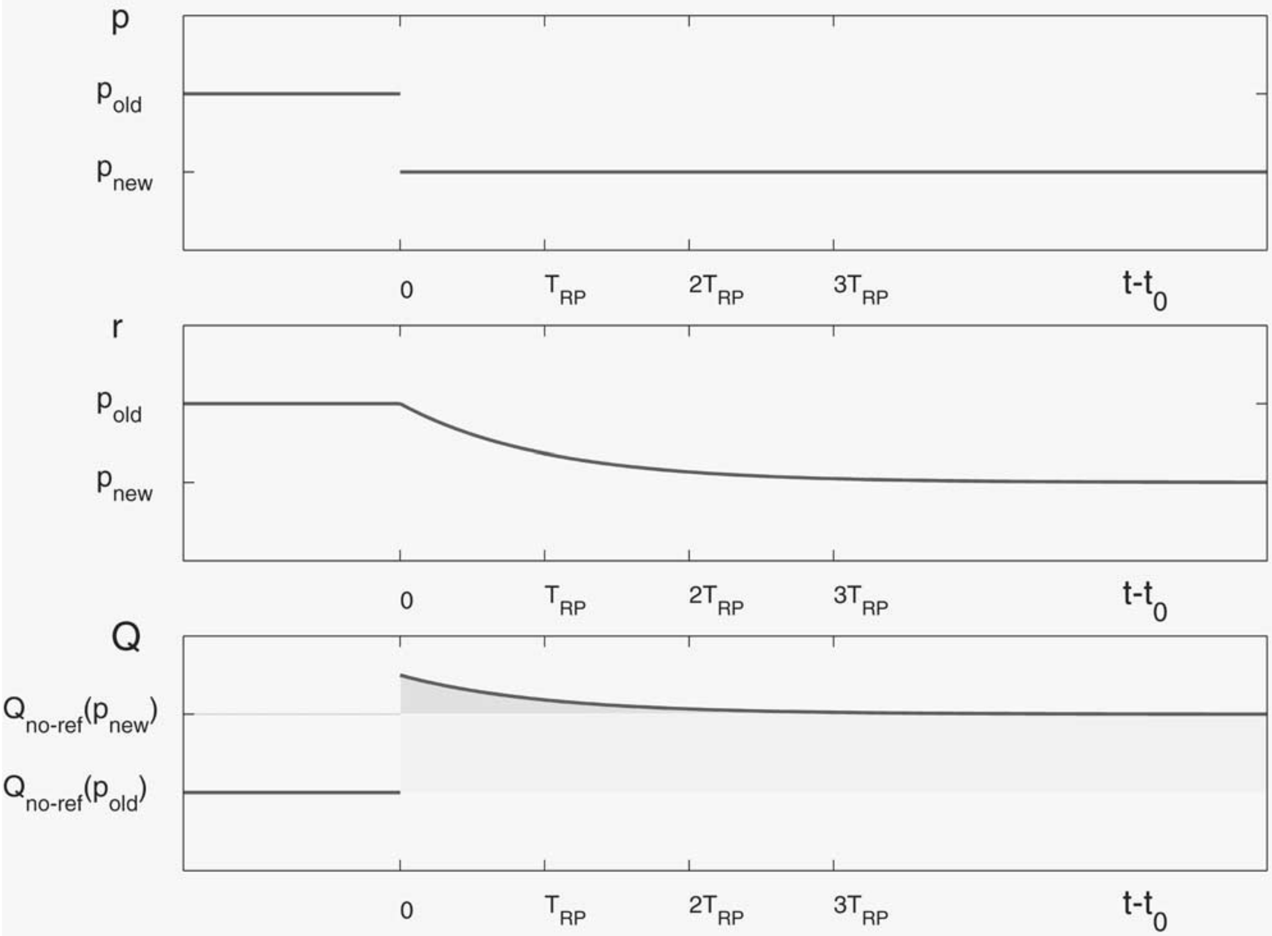
6.1. Reference Price Formation: Continuous Formulation

Following Sorger (1988) and Kopalle and Winer (1996), a reference price formation can be modeled using a continuous-time formulation:

$$r(t) = e^{-\beta(t-t_0)}\left[r_{t_0} + \beta\int_{t_0}^{t} e^{\beta(s-t_0)}p(s)\,ds\right] \quad t_0 \le t, \tag{12}$$

where $r_{t_0} := r(t_0)$ is the reference price at time t_0, and β is the continuous "memory" parameter. As we shall see, it is convenient to replace β with

FIGURE 1
Dynamics of Price (Equation (13)), Reference Price (Equation (14)), and Demand (Equation (15))



$$T_{RP} = \frac{1}{\beta},$$

which is the characteristic time scale for reference price effects (see Section 7).

As with the discrete formulation, we begin by considering the case of a single price change at time t_0, that is,

$$p(t) = \begin{cases} p_{old} & t < t_0 \\ p_{new} & t \ge t_0 \end{cases}. \tag{13}$$

In this case, reference price is equal to

$$r(t) = \begin{cases} p_{old} & t < t_0 \\ p_{new} + (p_{old} - p_{new})e^{-(t-t_o)/T_{RP}} & t \ge t_0 \end{cases}, \tag{14}$$

and demand is given by

$$Q(t) = \begin{cases} Q_{no-ref}(p_{old}) & t < t_0 \\ Q_{no-ref}(p_{new}) - \gamma(p_{old} - p_{new})e^{-(t-t_o)/T_{RP}} & t \ge t_0 \end{cases}. \tag{15}$$

The dynamics of $p(t)$, $r(t)$, and $Q(t)$ are shown in Figure 1, where the lighter and darker gray areas represent the contribution of price and reference price to the change in demand, respectively. We can see that T_{RP} is the characteristic time scale for reference price effects; that is, T_{TP} represents the time frame (days/weeks/months) for the new price p_{new} to take hold as the new reference price and for the reference price effect on demand to disappear.

6.2. Price Elasticity of Demand in the Presence of Reference Price Effects (Continuous Approach)

When reference price effects on demand are added to those of the actual price (3), we note that by (15),

$$\frac{\partial Q}{\partial p}(p_{old}) = \lim_{p_{new}\to p_{old}} \frac{Q(p_{new})-Q(p_{old})}{p_{new}-p_{old}} = Q'_{no-ref}(p_{old}) - \gamma e^{-(t-t_o)/T_{RP}}, \quad t_0 \le t. \tag{16}$$

Combining (1) and (16), we have that the price elasticity of demand in the presence of reference price effects is given by[6]

$$\varepsilon = [Q'_{no-ref}(p) - \gamma e^{-(t-t_0)/T_{RP}}]\frac{p}{Q_{no-ref}(p)}. \tag{17}$$

For example, when demand function is linear in p (equation (10)), ε is given by

$$\varepsilon = -[\delta + \gamma e^{-(t-t_0)/T_{RP}}]\frac{p}{a-\delta p}. \tag{18}$$

These expressions for ε show that price elasticity is also dependent on the time that elapsed from the price change at which the new demand level is evaluated and not just on p:

$$\varepsilon = \varepsilon(p;\, t - t_0). \tag{19}$$

We note that the first and second terms in the brackets in expressions (17) and (18) correspond to the lighter and darker gray areas in Figure 1, respectively.

To better understand the time dependence in the expression for elasticity, we recall that elasticity describes the response of demand to a price change, that is,

$$\varepsilon \approx \left(\frac{Q_{new}-Q_{old}}{p_{new}-p_{old}}\right)\left(\frac{p_{old}}{Q_{old}}\right).$$

Here, p_{old} and Q_{old} are price and quantity demanded before the price change, and p_{new} is price after the price change. However, because of reference price effects, demand after a price change varies over time, that is,

$$Q_{new} = Q_{new}(t - t_0).$$

It is because of this t dependence of Q_{new} that ε is time dependent. Therefore, $\varepsilon(p, t-t_0)$ is a measure of the change between demand levels just before the price change, and t time units after the price change.

Definition (17) is a general form for the price elasticity of demand in the presence of reference price effects. The expressions derived earlier, using the discrete formulation, correspond to the two extreme cases of (17). For example, elasticity immediately after a price change (i.e., $t \to t_0+$) is equal to the (discrete) immediate-term elasticity:

$$\lim_{t\to t_0^+} \varepsilon(p;\, t-t_o) = \varepsilon_{immediate-term}.$$

Similarly, elasticity in a sufficient length of time after the price change (i.e., $t - t_0 \to \infty$) is equal to the (discrete) long-term elasticity:

$$\lim_{t\to\infty} \varepsilon(p;\, t-t_o) = \varepsilon_{long-term}.$$

In addition, definition (17) shows that the time scale for immediate and long term is given by T_{RP} since

$$\varepsilon \begin{cases} \varepsilon_{immediate-term} & 0 \le t - t_0 << T_{RP} \\ \varepsilon_{long-term} & t - t_o >> T_{RP} \end{cases}.$$

Therefore, $\varepsilon_{immediate\text{-}term}$ characterizes the impact of a price change on demand during price promotions, while $\varepsilon_{long\text{-}term}$ characterizes the long-term impact of a price change on demand.

7. TIME SCALE FOR REFERENCE PRICE EFFECTS

To quantify the terms *immediate term* and *long term*, we need to estimate the continuous memory parameter β. Empirical studies, however, typically estimate the discrete memory parameter η using (2). Therefore, we now derive the relation between the discrete formulation of reference price (2) and the continuous one (12).

From equation (12), we have that

$$r(t_n) - e^{-\beta(t_n - t_{n-1})} r(t_{n-1}) = (1 - \beta e^{-\beta(t_n - t_{n-1})}) p(t_{n-1}), \tag{20}$$

where $t_n - t_{n-1} \approx T_{interpurchase}$, the average time duration between consecutive purchases in a product category. The T_{RP}, therefore, is a category-specific and not a brand-specific construct.[7]

As a result, equation (20) can be rewritten as

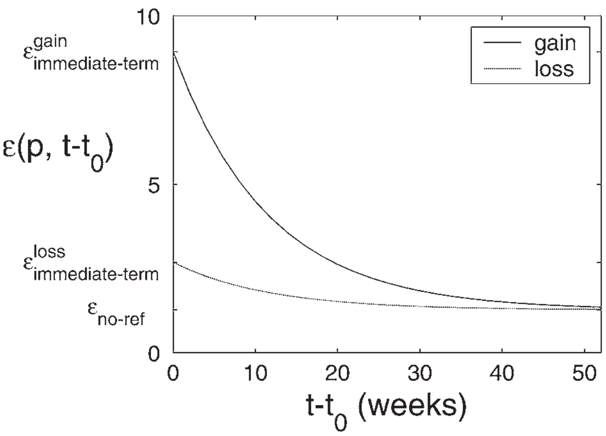
$$r_n = e^{-\beta T_{interpurchase}} r_{n-1} + (1 - e^{-\beta T_{interpurchase}}) p_{n-1}.$$

Comparison of this relation with (2) gives

$$\beta = \frac{\ln(1/\eta)}{T_{interpurchase}} \text{ and } T_{RP} = \frac{T_{interpurchase}}{\ln(1/\eta)}. \tag{21}$$

T_{RP} is the characteristic times scale for reference price effects. A good analogy for T_{RP} would be the half lifetime of radioactive materials. Thus, if the half lifetime of a material is 1 week, then after 1 day, almost no material would be lost, whereas after 4 weeks, most of the material would have disintegrated. Similarly, if the T_{RP} of a product is, say, 5 weeks, then marketers can conclude that 1 week after a price change, consumers would not have time to "absorb" the new price, whereas after 15 weeks, consumers would have fully adopted the new price as their new

FIGURE 2
$\varepsilon(p, t - t_0)$ for Peanut Butter



reference price. In that sense, T_{RP} is the characteristic time scale for reference price updating.

Going back to the peanut butter example and applying relation (21) with $\eta = 0.47$ and $T_{interpurchase} = 8.7$ weeks (Briesch, Krishnamurthi, Mazumdar, and Raj 1997), we get

$$T_{RP}(\textit{peanut butter}) = \frac{8.7\ \textit{weeks}}{\ln(1/0.47)} \approx 11.5\ \textit{weeks}.$$

8. EMPIRICAL ILLUSTRATION: CONTINUOUS FORMULATION

Having calculated that $T_{RP}(\textit{peanut butter}) \approx 11.5$ weeks, we can apply the continuous formulation approach to the example of Section 5. In Figure 2, we plot the t dependence of $\varepsilon(p, t - t_0)$ for peanut butter, evaluated at $p = p_{av}$. As can be seen, $\varepsilon(p, t - t_0)$ decreases monotonically from $\varepsilon_{immediate\text{-}term}$ to $\varepsilon_{no\text{-}ref}$. In addition, $\varepsilon_{immediate\text{-}term}$ is a good approximation to $\varepsilon(p, t - t_0)$ during roughly the first 5 weeks after the price change (i.e., $0 \le t - t_0 \le T_{RP}/2$). After about 20 weeks (i.e., $t - t_0 \ge 2\ T_{RP}$), $\varepsilon(p, t - t_0)$ is well approximated by $\varepsilon_{no\text{-}ref}$.

In a study that explored the short- and long-term effects of advertising and price promotion on consumers' price sensitivity, Mela, Gupta, and Lehmann (1997) defined a time frame of less than 4 weeks as short term and a quarter of a year as long term. This identification did not consider the variation between product categories or consumer characteristics. Our analytical approach enables us to build on their study and provide a rich and more insightful identification for these time-dependent effects.

Recall that the time scale for reference price effects, T_{RP}, is dependent on the product category (i.e., exposure to price information that is related to the frequency of purchasing this category, $T_{interpurchase}$) and consumers' memory capability (i.e., decaying over time, η). We can obtain a more realistic identification of the short- and long-term effects of price on consumer behavior. We used data from Briesch et al. (1997) to calculate the relevant time intervals (i.e., $t < T_{RP}/2$ for short term and $t > 2T_{RP}$ for long term) for different product categories. These results are presented in Table 1.

9. IMPLICATION TO ELASTICITY MEASUREMENT

The empirical illustration in Sections 5 and 8 shows that the effect of the reference price on the immediate-term price elasticity of demand can be quite large, and immediate-term price elasticity is substantially different from regular elasticity. Empirical measurements of price elasticity of demand would, therefore, be more accurate if they were to distinguish between the two and measure each one "separately."

Ideally, that can be done by first estimating the demand function in the presence of the reference price effects from empirical data (e.g., equation (11) for peanut butter) and then using the demand function to estimate immediate- and long-term price elasticities from equations (6) and (8), as was done in Section 5. This approach, however, is much more demanding than standard elasticity estimates. Therefore, one can adopt an alternative approach in which the data points are separated into two groups, those shortly after a price change and those long after it. With this approach, one can estimate immediate- and long-term changes in demand with, for example,

$$Q_{immediate-term} \approx average\{Q(t_i)\}_{0 < t_i - t_0 \le T_{RP}/2},$$
$$Q_{long-term} \approx average\{Q(t_i)\}_{2T_{RP} < t_i - t_0}.$$

The corresponding elasticity estimates are given by

$$\varepsilon_{immediate-term} \approx \left(\frac{Q_{immediate-term} - Q_{old}}{p_{new} - p_{old}}\right)\left(\frac{p_{old}}{Q_{old}}\right),$$
$$\varepsilon_{no-ref} \approx \left(\frac{Q_{long-term} - Q_{old}}{p_{new} - p_{old}}\right)\left(\frac{p_{old}}{Q_{old}}\right).$$

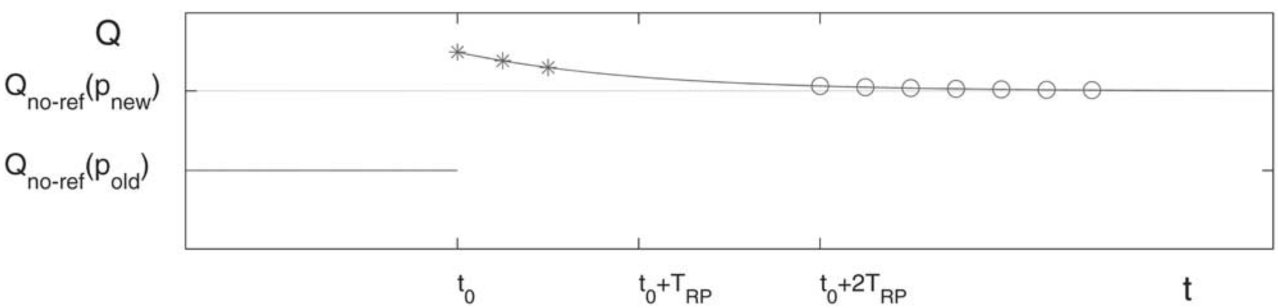
If we apply this approach to the peanut butter data by calculating the demand over a time period of 45 weeks from the price change and sampling it every day, we get

$$\varepsilon^{gain}_{immediate-term} \approx 7.3,\ \varepsilon^{loss}_{immediate-term} \approx 2.4,\ \varepsilon_{no-ref} \approx 1.7.$$

TABLE 1
Identification of Short- and Long-Term Price Effects for Various Product Categories

Category	$T_{interpurchase}$ *(Weeks)*	η	T_{RP} *(Weeks)*	*Short Term (Weeks)*	*Long Term (Weeks)*
Liquid detergent	11.40	0.57	20.28	10.14	40.56
Tissue	5.20	0.65	12.07	6.04	24.14
Peanut butter	8.70	0.47	11.52	5.76	23.04
Ground coffee	6.50	0.57	11.56	5.78	23.12

FIGURE 3
Separation of Data Points Used for Estimating Immediate-Term (*) and Long-Term (o) Elasticities



These estimates are relatively close to the values calculated in Section 5 and capture the differences between immediate- and long-term elasticities, as well as between gain and losses (see Figure 3).

10. COMPETITION

Our analysis can be extended to competitive situations. We use a demand function with an additive form for prices and brand-specific reference price effects.[8] For simplicity, we discuss the duopoly case (i.e., Firm A and Firm B) in which demand for the product of Firm A is given by

$$Q^A = Q^A_{no-ref}(p^A, p^B) - \gamma(p^A - r^A).$$

Here, Q^A_{no-ref} is market demand in the absence of reference price effects for the product of Firm A; p^A and p^B are prices of products A and B, respectively; and r^A is the reference price of product A. In this case, following the derivation of equation (17), the price elasticity of demand for the product of Firm A is given by

$$\varepsilon^A(p^A, p^B, t - t_o) = \frac{\frac{\partial Q^A}{\partial p^A}}{\frac{Q^A}{p^A}} = \left[\frac{\partial Q^A_{no-ref}}{\partial p^A} - \gamma e^{-(t-t_o)/T_{RP}}\right]\frac{p^A}{Q^A_{no-ref}}.$$

Therefore, the effect of the reference price on the price elasticity of demand is the same as in the monopoly case:

$$\varepsilon^A_{immediate-term} \approx \left[\frac{\partial Q^A_{no-ref}}{\partial p^A} - \gamma\right]\frac{p^A}{Q^A_{no-ref}},$$

$$\varepsilon^A_{long-term} \approx \frac{\partial Q^A_{no-ref}}{\partial p^A}\frac{p^A}{Q^A_{no-ref}} = \varepsilon^A_{no-ref}$$

and

$$RCIE = \frac{\varepsilon^A_{immediate-term} - \varepsilon^A_{no-ref}}{\varepsilon^A_{no-ref}} = \frac{\gamma}{-\frac{\partial Q^A_{no-ref}}{\partial p^A}} = \frac{\text{effect of } (p^A - r^A) \text{ on demand}}{\text{effect of } p^A \text{on demand}}.$$

Consequently, we see that reference price effects on the price elasticity of demand in a competitive situation are the same as in the monopoly case. There is, of course, a difference between these two cases, which is manifested in ε_{no-ref} through the P^B effect on Q^A_{no-ref}.

Thus, the effects of the reference price are the result of an own-brand activity, whereas other firms' price-related activities are captured by the price component of the demand function and are reflected through a lower demand for Firm A's product (when P^B is decreasing) and a higher level of demand (when P^B is increasing). The resulting competitive effect is reflected, therefore, in capturing sales from competitors.

11. MULTIPLE PRICE CHANGES

The expressions for immediate-term elasticity (equation (8)) and for elasticity with reference price effects (equation (17)) were derived for the case of a *single* price change. In this case, prior to the price change, the reference price is equal to the market price, and demand is given by $Q_{no\text{-}ref}(p_{old})$. However, if there were earlier price changes, demand just before the price change is given by $Q_{old} = Q_{no\text{-}ref}(p_{old}) - \gamma(p_{old} - r)$, where r, the reference price at the time of the price change, can be different from p_{old}. As a result, the expressions for immediate-term elasticity (equation (8)) and for elasticity with reference price effects (equation (17)) are, more generally, given by

$$\varepsilon_{immediate-term}(p,r) = [Q'_{no-ref}(p) - \gamma]\frac{p}{Q_{no-ref}(p) - \gamma(p-r)} \tag{22}$$

and

$$\varepsilon(p,r,t-t_0) = [Q'_{no-ref}(p) - \gamma e^{-(t-t_0)/T_{RP}}]\frac{p}{Q_{no-ref}(p) - \gamma(p-r)}, \tag{23}$$

respectively. However, when the previous price change occurs more than $2T_{RP}$ time units before the current change, one can safely assume that $r \approx p_{old}$ and use expressions (8) and (17). Therefore, one needs to work with the modified expressions (22) and (23) only when the previous price change occurs less than $2T_{RP}$ time units. In that case, although the calculations are somewhat more complex, the overall scenario remains the same.

At the extreme case of unstable prices, reference price effects on demand will still be determined by the exposure time and depth of price changes. That is, if there is a continuous monotonic price change, consumers will not have enough time to adjust to the new price level. The resulting effect on price elasticity is, therefore, depending on the T_{RP} for the product category. If price changes are made in very short time intervals, $<< T_{RP}/2$, then consumers will not absorb the new price each time it has been changed and will consider the aggregate change as a single price change. As for the $Q_{no\text{-}ref}$ component, it can be estimated through a demand function that ignores reference price effects. The multiple price changes case has an effect only on the stability of the effects of the reference price on demand and elasticity.

12. PROMOTIONAL ELASTICITY

Blattberg and Neslin (1989) postulated that the large increase in promotional elasticity, compared with regular elasticity, is primarily due to (1) brand switching by consumers, (2) inventory behavior (stockpiling), and (3) transaction utility effects (i.e., the sense of "gain"; Thaler 1985). By combining household-level data, it was estimated that approximately 80 percent of this increase is attributed to brand switchers (Blattberg and Neslin 1989; Gupta 1988).

In the case of a monopoly, brand-switching effects do not exist. In addition, inventory behavior has an effect on demand only after the consumer has already made at least one purchase during the promotion. Since T_{RP} is roughly equal to the interpurchase time of the product (equation (21)), the effect of inventory behavior is small in the immediate term and only gains importance in the intermediate term when $t - t_0 = O(T_{RP})$. Therefore, the increase in demand during the promotional activity of a monopoly is due mostly to transaction utility effects (dark gray area in Figure 1), promotional elasticity is roughly equal to immediate-term elasticity, and

$$\frac{\varepsilon_{promotion} - \varepsilon_{no-ref}}{\varepsilon_{no-ref}} \approx \frac{\gamma}{-Q'_{no-ref}(p)}.$$

In a competitive environment, where brand switching does occur, the above relation gives the relative contribution of transaction utility to the increase in promotional elasticity, and it accounts for some of the "missing 20 percent."

13. HETEROGENEITY

Our analysis so far has treated the market as one group of homogeneous consumers. The literature, however, suggests that different consumers might have a different reference price in mind. They also might have different evaluations of gains and losses. To capture such heterogeneity and examine whether it affects our modeling results, we introduce a heterogeneity analysis. We derive this analysis by allowing different memory parameters, β, and different weights of gains and losses, γ, for different groups of consumers.

We consider a market with n groups of different consumers with different demand function parameters. Each of the n groups has a certain proportion in the population, w_i, $i = 1, 2, \ldots, n$, such that $\sum_{i=1}^{n} w_i = 1$. If each consumer has a different reference price (i.e., n groups of size = 1), then $w_i = 1/n$. Similarly, let us define the reference price for each group as r_i, $i = 1, \ldots, n$. The demand for each group i is, therefore, given by

$$Q_i(p) = Q^i_{no-ref} - \gamma_i(p - r_i).$$

The resulting aggregate demand is given by

$$Q(p) = \sum_{i=1}^{n} w_i Q_i(p) = Q_{no-ref}(p) - \sum_{i=1}^{n} w_i \gamma_i (p - r_i),$$

where $Q_{no-ref}(p) = \sum_{i=1}^{n} w_i Q^i_{no-ref}$. Let us also define $\gamma = \sum_{i=1}^{n} w_i \gamma_i$ and the resulting weighted reference price as $r = \sum_{i=i}^{n} \frac{w_i \gamma_i}{\gamma} r_i$. This formulation yields the same demand function structure that was obtained in the homogeneous analysis earlier, $Q(p) = Q_{no-ref} - \gamma(p - r)$. Consequently, the discrete analysis results hold also for the heterogeneity case. In the continuous case, however, a new notation is needed. Let us define the reference price as

$$r_i(t) = e^{-\beta_i (t - t_0)} \left[r^i_{t_0} + \beta_i \int_{t_0}^{t} e^{\beta_i (s - t_0)} p(s)\, ds \right], \qquad t_0 \le t, i = 1,2\ldots,n \tag{24}$$

where $r^i_{t_0} := r(t_0), i = 1, \ldots, n$ is the reference price at time t_0 (i.e., assumed to be equal for all types), and β_i is the continuous "memory" parameter for consumer type i. Also, let

$$T^i_{RP} = \frac{1}{\beta_i}, \qquad i = 1, 2, \ldots, n$$

be the characteristic time scale for consumer group i. Similar to equation (15) and employing the above arguments, we have

$$Q(t) = \begin{cases} Q_{no-ref}(p_{old}) & t < t_0 \\ Q_{no-ref}(p_{new}) - \gamma(p_{old} - p_{new}) \sum_{i=1}^{n} \frac{w_i \gamma_i}{\gamma} e^{-(t - t_0)/T^i_{RP}} & t \ge t_0 \end{cases}. \tag{25}$$

The resulting price elasticity is similar to the one obtained in the general analysis in equation (17):

$$\varepsilon = \left[Q'_{no-ref}(p) - \sum_{i=1}^{n} w_i \gamma_i e^{-(t - t_0)/T^i_{RP}} \right] \frac{p}{Q_{no-ref}(p)}. \tag{26}$$

It is easy to verify that except for the additional notations, there are no qualitative changes between the results obtained in the heterogeneity case (equation (26)) and the general case (equation (17)). In sum, the general results also hold for the case of consumers' heterogeneity in the reference price.

14. FINAL REMARKS

The theoretical distinction between immediate- and long-term elasticities has implications for the methodology of estimating the price elasticity of demand. Specifically, measurements made shortly after price changes should be used to determine short-term elasticity, while those made a sufficient period of time after the price change should be used to determine long-term elasticity (Section 9). The exact duration to determine these time periods, T_{RP}, is product specific and can be estimated using relation (21) and the values of η and $T_{interpurchase}$ for the product category.

This distinction has several practical implications as well, by emphasizing the advantage of (1) using immediate-term price elasticity values when the planned price change duration $< T_{RP}$ and (2) using long-term (regular) price elasticity values when the price change duration $> T_{RP}$.

An interesting result, which is counterintuitive to classical economic theory about price elasticity changes over time, predicts low-price elasticity in the short term due to relatively high inflexibility in the short term (e.g., number of substitutes) and lower price elasticity in the long term. Our results indicate an opposite direction, with higher elasticities in the short term than in the long term. It should be noted, however, that we did not account for other external factors that might have an effect on elasticity, except for competition. It would be interesting to further explore this issue by adding other quantity-demanded related factors to the demand function and determine the short- and long-term elasticities.

In this article, we have focused on the effect of the reference price on the price elasticity of demand. Our quantitative methodology, however, can be applied to other effects that depend on the time from the price change, such as stockpiling.

Another direction for further research that is yielded from our analytical approach is the possibility of incorporating the time dependence of reference price effects into consumer behavior models. Let's start with a basic formulation of a deterministic utility component of a multinomial logit choice model that is commonly used in capturing reference price effects in brand choice studies. For simplicity, we assume only reference price effects on consumers' choice,

$$V_{ikn} = Gain(p_{ikn} - r_{ikn})\alpha_1 + Loss(p_{ikn} - r_{ikn})\alpha_2,$$

where *Gain* and *Loss* are the gain and loss parameters, and α is an indication function for positive and negative deviations between the actual price, p, that consumer i has observed for brand k at purchase occasion n and the reference price, r, of consumer i for brand k at purchase occasion n as follows:

$$\alpha_1 = \begin{cases} 0 & p_{ikn} \le r_{ikn} \\ 1 & p_{ikn} > r_{ikn} \end{cases} \quad \text{and} \quad \alpha_2 = \begin{cases} 1 & p_{ikn} \le r_{ikn} \\ 0 & p_{ikn} > r_{ikn} \end{cases}.$$

Now, we can introduce the short-, medium-, and long-term effects of the reference price into this model in the following way:

$$\begin{aligned} V_{ikt} = & Gain_{short}(p_{ikn} - r_{ikn})\alpha_1 \xi_S \\ & + Gain_{medium}(p_{ikn} - r_{ikn})\alpha_1 \xi_M \\ & + Gain_{long}(p_{ikn} - r_{ikn})\alpha_1 \xi_L \\ & + Loss_{short}(p_{ikn} - r_{ikn})\alpha_2 \xi_S \\ & + Loss_{medium}(p_{ikn} - r_{ikn})\alpha_2 \xi_M \\ & + Loss_{long}(p_{ikn} - r_{ikn})\alpha_2 \xi_L , \end{aligned}$$

where $Gain_{short}$, $Gain_{medium}$, and $Gain_{long}$ are the gain parameters for the short-, medium-, and long-term effects. $Loss_{short}$, $Loss_{medium}$, and $Loss_{long}$ are the loss parameters for the short-, medium-, and long-term effects, and ξ is an indication function for the time duration, *t*, that elapsed from the last price change until the *n*th purchase as follows:

$$\xi_S = \begin{cases} 1 & t \le T_{RP}/2 , \\ 0 & otherwise \end{cases}$$

$$\xi_M = \begin{cases} 1 & T_{RP}/2 < t \le 2T_{RP} \\ 0 & otherwise \end{cases} ,$$

$$\text{and} \quad \xi_L = \begin{cases} 1 & t > 2T_{RP} \\ 0 & otherwise \end{cases},$$

where ξ_S, ξ_M, and ξ_L represent the short-, medium-, and long-term time periods, respectively. Such model formulation can account for the time dependence of reference price effects.

Our approach can be extended to include other types of competition formulations, as well as reference price conceptualizations that are not brand specific but rather category specific (e.g., Hardie, Johnson, and Fader 1993). Another venue for future research can be the inclusion of nonprice variables that might have an effect on the price elasticity of demand.

To conclude, this article has addressed two main issues: analytical formulation of the price elasticity of demand in the presence of asymmetric reference price effects and identification of the time dependence of these effects on consumer behavior (i.e., short- and long-term effects). The resulting implications from this approach are that price elasticity is very sensitive to the time that has elapsed since the price change, better estimations of elasticity can be derived when the time dependence is accounted for, and new empirical modeling formulations can be developed to obtain better estimates of price effects.

NOTES

1. The demand function is an aggregation of all individual consumers' demand. A modified Klein-Rubin utility function can be used that allows for exact linear aggregation when reference price effects are introduced (see Putler 1992).

2. Precise definitions of *short term* and *long term* are provided in Section 6.2.

3. We assume here that price was held constant for a "sufficiently long time" before the price change (see Section 11).

4. We used the average price for elasticity calculations as it is commonly used for such purposes (see, e.g., Tellis 1988).

5. This value is in the range of the Tellis (1988) meta-analysis findings about the price elasticity of demand.

6. See Section 11 for the case of multiple price changes.

7. Our modeling approach assumes that consumers update their reference price whenever they are exposed to price information, rather than when the price is changed or when they actually make a purchase. Since, however, it is easier to measure the interpurchase time than the interexposure time, we use the former as a proxy to the latter.

8. A detailed description of such demand function can be found in Putler (1992) and Kopalle, Rao, and Assunção (1996).

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