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Optimal Price Promotion in the Presence of Asymmetric Reference-price Effects

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In this study we demonstrate how a reference price may affect the degree of price rigidity/flexibility. For this, we construct a model of reference-price formation, which we use to analyze the effect of asymmetric reference price (cut 'effects') on the profitability of price promotions. We derive explicit expressions for the additional profits earned during a promotional period due to consumer perception of a 'gain', and for the post-promotion loss of potential profits due to consumer perception of a 'loss'. We show that when effects of losses on demand are greater than effects of gains ('loss aversion'), price promotions always lead to a decline in profits. When, however, effects of gains are larger than those of losses, price promotions, as well as reverse price promotions (i.e. price increase) can be profitable. In the latter case we calculate the optimal depth and duration of a price promotion. We also show that reference price can affect price rigidity and flexibility.

INTRODUCTION

Price rigidity and flexibility play an important role in macroeconomics, industrial engineering and marketing. While several theories have been suggested to explain price rigidity (Blinder *et al.*, 1998), the role that asymmetric reference-price effects play in rigidity or flexibility of prices has not been addressed. As we shall see in this study, reference price can be yet another reason for possible rigidity of prices. An additional motivation for this study is to explore the role of reference price in price promotions. Price promotion is a common managerial practice employed for a variety of reasons such as luring customers into stores to buy other products at regular price (i.e., loss leader), increasing repeat buying, increasing market share among brand switchers and targeting deal-prone consumers. Although there is a large body of literature on the effects of price promotions on demand and profitability of firms, relatively little has been done to explore the effect of promotional activities on the reference price of a specific brand (Lattin and Bucklin, 1989). Specifically, the concept that both deal frequency and depth of price cut can affect reference price has gained virtually no attention by researchers (Kalwani and Yim, 1992).

To illustrate this problem, let us consider a retailer who promotes a certain product 'too often'. In that case, consumers would adopt the promoted price as the 'new regular' price and consider the 'old regular' price as too expensive. Thus, although the retailer would reap profits during the price-promotion periods, in the long run this policy would lower the product's reference price in consumer evaluations, resulting in reduced profits during the nonpromotional periods

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(Blattberg *et al.*, 1995). Therefore, retailers planning price promotions should consider both the increased profits during promotions and the reduced profits during post-promotion periods. However, for this to be feasible, retailers should be able to quantify the impact of the duration of the promotion and the depth of the price cut on reference price and profitability. Since most of the marketing and economics literature on this subject is empirical in nature, our goal is to develop a theoretical tool that would enable retailers to determine the profitability of a price promotion in the presence of reference-price effects, and hence, the optimal depth duration and of a price promotion.

Our starting point is a standard mathematical model for reference-price effects. Unlike previous studies, in the calculations we separate price and reference-price effects on profits and derive separate expressions for the promotion period (Proposition 1) and for the post-promotion period (Proposition 2). Combining these expressions, we obtain the reference price component of the overall profits $\Delta\Pi_{ref}$ (Proposition 4). Since the expression for $\Delta\Pi_{ref}$ includes only reference-price effects on profits, it allows retailers to quantify the impact of reference price on profitability, even in scenarios more complex than those covered in our model. In other words, retailers can 'ignore' reference-price effects in their calculations of the profitability of a price promotion, and then include these effects by simply adding the value of $\Delta\Pi_{ref}$.

Our calculations can also be used to gain insight into the relationship between asymmetry of reference-price effects and the profitability of price promotions. The main conclusion of this study is that when the effects of losses on demand are greater than the effects of gains, price promotions always lead to a decline in profits (Proposition 6). Thus, the additional profits gained during the promotion are lower than the reduced profits during the post-promotion period. When, however, the effects of gains are larger than the effects of losses, price promotions, as well as reverse promotions (i.e., price increase), can be profitable provided that the correct promotion price and duration is chosen. In that case, we show how to calculate the optimal promotion price and duration (Proposition 8). Perhaps surprisingly, we find that the optimal reverse promotion (i.e., a price increase) is slightly more profitable than the optimal price promotion (Proposition 9). In order to simplify the presentation, we begin by analyzing the profitability of a single price promotion for a monopolistic retailer. Our modeling approach, however, can be extended to more realistic situations, such as multiple price changes, heterogeneity among consumers with respect to reference price, and competition (see Appendix). In the section 'Price Rigidity' we discuss the role of reference-price effects in price rigidity and flexibility. Concluding remarks are given in the end.

REFERENCE PRICE AND PRICE PROMOTION

Reference price can be defined as an internal price that consumers compare with the actual shelf price to evaluate whether an observed price is high or low. When the price is higher than the reference price, consumers have the feeling of a 'loss', resulting in reduced market demand for the product. When, however, the price is lower than the reference price, the feeling of a 'gain' results in increased market demand. These gains and losses are not evenly evaluated by consumers (Kahneman and Tversky, 1979). Empirical findings in the literature are primarily concerned with a choice context and support the notion that consumers behave as if they possess a reference price in their evaluations (Kalyanaram and Winer, 1995).

Price promotion is an important managerial tool as a means of attracting customers to increase store traffic, given the fact that most customers purchase other products at regular price (Mulhern and Padgett, 1995). The key elements that characterize price promotions are the depth, duration and frequency of the price cut. For example, Kumar and Pereira (1995) found that the frequency and timing of price promotions affect the short-term response of the firm's sales. Since price promotions are more important than other promotional tools (Farris and Quelch, 1987), there is a need to fully explore their potential negative impact on the demand for a firm's brand after the promotion has ended.

THE MODEL

Reference price is formed through consumer exposure to the product price when shopping. We use the standard model for reference-price

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formation, where reference price is an exponentially weighted average of past prices (Kopalle and Winer, 1996):

$$r(t) = e^{-\beta(t-t_0)}\left[r_{t_0} + \beta\int_{t_0}^{t} e^{\beta(s-t_0)}p(s)\,ds\right], \quad t_0 \leqslant t, \quad (1)$$

where $r_{t_0} := r(t_0)$ is the reference price at time $t = t_0$ and β is the 'memory' parameter.

Let us denote the demand in the absence of reference-price effects by $Q_{no\text{-}ref}(p)$. We assume that the effect of reference price on demand is additive and separable (Greenleaf, 1995; Kopalle *et al.*, 1996; Kopalle and Winer, 1996), i.e.,

$$Q(p,r) = Q_{no\text{-}ref}(p) - \gamma\cdot(p-r). \quad (2)$$

In order to capture the differential effect of losses and gains on consumer demand (Kahneman and Tversky, 1979), we use the *asymmetric* model where

$$\gamma = \begin{cases} \gamma_g, & p \leqslant r \text{ (gain)}, \\ \gamma_l, & p > r \text{ (loss)}. \end{cases} \quad (3)$$

Both γ_g and γ_l are positive, ensuring that when $p > r$ demand decreases and vice versa.

Most empirical studies at the disaggregate level show that consumers are loss aversive (e.g., Winer, 1986; Lattin and Bucklin, 1989; Kalyanaram and Winer, 1995). However, other studies at the aggregate level (Greenleaf, 1995) and the disaggregate level (Murthi and Kalyanaram, 1999) suggest that gain effect is greater than loss effect. Consequently, we consider both cases in this study.

PROFITABILITY OF A SINGLE PRICE PROMOTION

Let us consider the following two strategies.

Strategy I (no promotion):
The retailer maintains a constant price p_1 throughout the planning interval, i.e., $p^I(t) \equiv p_1$ for $0 \leqslant t < \infty$.

Strategy II (single promotion):
The retailer maintains a constant price p_1 throughout the planning interval, except for a single price promotion which begins at t_0 and lasts T time units, during which the price is p_2, i.e.,

$$p^{II}(t) = \begin{cases} p_1, & 0 \leqslant t < t_0 \text{ (before promotion)}, \\ p_2, & t_0 \leqslant t < t_0 + T \text{ (promotion)}, \\ p_1, & t_0 + T \leqslant t < \infty \text{ (post-promotion)}. \end{cases}$$

The price p_2 may be lower (promotion) or higher (reverse promotion) than p_1.

Although Strategy II is not the optimal strategy (see later sections), comparison of these two strategies will allow us to analyze the additional profits that result from a single price promotion.

To simplify the presentation, we focus on the case of a monopoly. We also assume that the retailer holds the 'regular' price p_1 constant for a sufficient length of time before the price promotion. Therefore, when the promotion starts consumer reference price is equal to p_1, i.e.,

$$r(t_0) = p_1. \quad (4)$$

We define the profitability of a single price promotion as the difference in overall profits between the two strategies. For a given price strategy $p^i(t)$, $i = I, II$, the profit-rate per unit of time is given by $\pi^i(t) = (p^i(t) - c)Q(p^i(t), r(t))$, where c is the production cost per unit. Therefore, *the difference in profits between the two strategies during the promotion period* $t_0 \leqslant t \leqslant t_0 + T$ is given by $\Delta\Pi^{promotion} = \int_{t_0}^{t_0+T} e^{-\alpha t}(\pi^{II}(t) - \pi^I(t))\,dt$, where α is the discounting factor. In order to separate price effects from reference-price effects, let us denote by $\pi_{no\text{-}ref} = Q_{no\text{-}ref}(p-c)$ the profit rate in the absence of reference-price effects. Therefore, *the difference in profits during the promotion period due to price effects* is given by $\Delta\Pi^{promotion}_{no\text{-}ref} = \int_{t_0}^{t_0+T} e^{-\alpha t}(\pi^{II}_{no\text{-}ref}(t) - \pi^{I}_{no\text{-}ref}(t))\,dt$. Let us denote by $\Delta\Pi^{promotion}_{ref}$ *the difference in profits during the promotion period due to reference-price effects.* Therefore, $\Delta\Pi^{promotion}_{ref} = \Delta\Pi^{promotion} - \Delta\Pi^{promotion}_{no\text{-}ref}$. We now calculate each of these terms explicitly.

Proposition 1:
Let (1)–(4) hold. Then, the difference in profits during the promotion period due to price effect is given by $\Delta\Pi^{promotion}_{no\text{-}ref} = e^{-\alpha t_0}[\pi_{no\text{-}ref}(p_2) - \pi_{no\text{-}ref}(p_1)](1 - e^{-\alpha T})/\alpha$. The difference in profits due to reference-price effect is given by

$$\Delta\Pi^{promotion}_{ref} = \begin{cases} \gamma_g e^{-\alpha t_0}(p_2 - c)(p_1 - p_2) \\ \quad\times\dfrac{1 - e^{-(\alpha+\beta)T}}{\alpha+\beta}, & p_2 \leqslant p_1, \\ \gamma_l e^{-\alpha t_0}(p_2 - c)(p_1 - p_2) \\ \quad\times\dfrac{1 - e^{-(\alpha+\beta)T}}{\alpha+\beta}, & p_2 > p_1. \end{cases}$$

Proof:
These results follow from the calculations in the Appendix. □

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During the post-promotion period $t_0 + T \leqslant t < \infty$, the price is the same under both strategies. Consequently, the difference in profits is only due to reference-price effects, i.e., $\Delta\Pi^{post\text{-}promotion} = \Delta\Pi_{ref}^{post\text{-}promotion}$. Based on the calculations in the Appendix, we can calculate this difference explicitly.

Proposition 2:
Let (1)–(4) hold. Then, the difference in profits between the two strategies during the post-promotion period is given by

$$\Delta\Pi^{post\text{-}promotion} = \begin{cases} -\gamma_l e^{-\alpha(t_0+T)}(p_1 - c)(p_1 - p_2) \\ \quad \times \dfrac{1 - e^{-\beta T}}{\alpha + \beta}, & p_2 \leqslant p_1, \\ -\gamma_g e^{-\alpha(t_0+T)}(p_1 - c)(p_1 - p_2) \\ \quad \times \dfrac{1 - e^{-\beta T}}{\alpha + \beta}, & p_2 > p_1. \end{cases}$$

From Propositions 1 and 2 we can immediately calculate the overall profitability of a price promotion $\Delta\Pi = \Delta\Pi^{promotion} + \Delta\Pi^{post\text{-}promotion}$. As before, it is instructive to separate the overall difference in profits between the two strategies into price effect and reference-price effect components: $\Delta\Pi = \Delta\Pi_{no\text{-}ref} + \Delta\Pi_{ref}$.

Proposition 3:
The overall difference in profits due to price effect is given by

$$\Delta\Pi_{no\text{-}ref} = -e^{-\alpha t_0}[\pi_{no\text{-}ref}(p_1) - \pi_{no\text{-}ref}(p_2)] \times \frac{1 - e^{-\alpha T}}{\alpha}.$$

When $p_2 < p_1$ (i.e., promotion), the overall difference in profits due to reference-price effect is given by $\Delta\Pi_{ref} = e^{-\alpha t_0}[\gamma_g(p_2 - c)(1 - e^{-(\alpha+\beta)T})/(\alpha + \beta) - \gamma_l(p_1 - c)(e^{-\alpha T} - e^{-(\alpha+\beta)T})/(\alpha + \beta)](p_1 - p_2)$. When $p_2 > p_1$ (i.e., reverse promotion), the overall difference in profits due to reference-price effect is given by $\Delta\Pi_{ref} = e^{-\alpha t_0}[\gamma_l(p_2 - c)(1 - e^{-(\alpha+\beta)T})/(\alpha+\beta) - \gamma_g(p_1 - c)(e^{-\alpha T} - e^{-(\alpha+\beta)T})/(\alpha+\beta)](p_1 - p_2)$.

In order to simplify the expressions in Proposition 3, let us note that the typical price promotion duration is short and, as a result, the discount factor, α only has a small impact on the results. In addition, in the case of frequently purchased goods we have that $\alpha \ll \beta$.[1] Therefore, we can use the approximations

$$\alpha T \approx 0, \quad \alpha + \beta \approx \beta, \quad \frac{1 - e^{-\alpha T}}{\alpha} \approx T \tag{5}$$

to simplify the expressions in Proposition 3:

Proposition 4:
Let (1)–(5) hold. Then, the overall difference in profits due to price effect is given by $\Delta\Pi_{no\text{-}ref} = -e^{-\alpha t_0}[\pi_{no\text{-}ref}(p_1) - \pi_{no\text{-}ref}(p_2)]T$. The overall difference in profits due to reference-price effect is given by

$$\Delta\Pi_{ref} = e^{-\alpha t_0}\frac{1 - e^{-\beta T}}{\beta} \times \begin{cases} [(\gamma_g - \gamma_l)(p_1 - c) - \gamma_g(p_1 - p_2)] \\ \quad \times (p_1 - p_2), & p_2 < p_1, \\ [(\gamma_g - \gamma_l)(p_1 - c) - \gamma_l(p_2 - p_1)] \\ \quad \times (p_2 - p_1), & p_2 > p_1. \end{cases}$$

Proposition 4 provides an *explicit expression for the reference price component of the overall profitability of price promotions*, i.e., $\Delta\Pi_{ref}$. Because this expression isolates the reference-price component of the profits, it can be used by managers trying to estimate the overall profitability of a price promotion even in more realistic settings, where additional factors (such as consumer deal proneness and switching tendency or the loss-leader effect) also play a role. We can also use the results of Proposition 4 to gain insight into the relationship between asymmetry of reference-price effects and profitability of price promotions. For example, from the explicit expression for $\Delta\Pi_{ref}$ it immediately follows that in the case of loss aversion, the loss of potential profits due to reference-price effects during the post-promotion period is always greater than the added profits due to reference-price effects during the promotion:

Proposition 5:
Let (1)–(5) hold, and assume that $\gamma_g \leqslant \gamma_l$. Then, $\Delta\Pi_{ref} < 0$ for all $p_2 \neq p_1$.

Thus, as far as reference-price effects on profits are concerned, a promotion can be profitable (i.e., $\Delta\Pi_{ref} > 0$) only when the effect of gains on demand is greater than that of losses. Note that

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this result holds for all values of the memory parameter β which satisfy assumptions (5), i.e., those for which $\beta \gg \alpha$.

When there are no reference-price effects, the optimal strategy is clearly to set p_1 to be the optimal price in the absence of reference price, i.e.,

$$p_1 = \arg\max_p \, \pi_{no\text{-}ref}(p). \tag{6}$$

In this case, profits always decrease during the promotion period, as p_2 is suboptimal, and thus $\Delta\Pi_{no\text{-}ref} < 0$ for all $p_2 \neq p_1$. Therefore, when p_1 is the optimal price (6), a promotion can only be profitable when $\Delta\Pi_{ref} > 0$. From Proposition 5, this can only occur when $\gamma_g > \gamma_l$. Thus, we have proven that when consumers are loss aversive, promotions are never profitable.

Proposition 6:
Let (1)–(5) hold, let p_1 be the optimal price (6) and let $\gamma_g \leqslant \gamma_l$. Then, no promotion is profitable, i.e., $\Delta\Pi < 0$ for all $p_2 \neq p_1$.

We can show that the inverse is also true, i.e., when $\gamma_g > \gamma_l$ there is always a promotion price $p_2 < p_1$ and an reverse promotion price $p_2 > p_1$ for which promotion and reverse promotion result in an overall increase in profits.

Proposition 7:
Let (1)–(5) hold, let p_1 be the optimal price (6), and let $\gamma_g > \gamma_l$. Then, there is always a promotion price $p_2 < p_1$ and an reverse-promotion price $p_2 > p_1$ for which promotion and reverse promotion are profitable, respectively.

Proof:
See Appendix.

OPTIMAL DEPTH AND DURATION OF PROMOTIONS

In Proposition 6 we saw that when p_1 is the optimal price and $\gamma_g > \gamma_l$, there are always a promotional price $p_2 < p_1$ and a reverse-promotion price $p_2 > p_1$ that would result in an increase in overall profits. Thus, when $\gamma_g > \gamma_l$, managers can utilize reference-price effects to increase their profits. A natural question is, thus, what is the optimal promotion price and duration. The optimal values can be found from the first order conditions

$$\frac{\partial \Delta\Pi[T, p_2]}{\partial T} = \frac{\partial \Delta\Pi[T, p_2]}{\partial p_2} = 0.$$

Using the expression for $\Delta\Pi$ from Proposition 4, we find that the optimal price promotion (i.e., $p_2 < p_1$) and its duration are the solution of:

$$\begin{aligned} &-[\pi_{no\text{-}ref}(p_1) - \pi_{no\text{-}ref}(p_2)] + [(\gamma_g - \gamma_l)(p_1 - c) \\ &\quad - \gamma_g(p_1 - p_2)](p_1 - p_2)\mathrm{e}^{-\beta T} = 0, \\ &\pi'_{no\text{-}ref}(p_2)T + [2\gamma_g(p_1 - p_2) - (\gamma_g - \gamma_l)(p_1 - c)] \\ &\quad \times \frac{1 - \mathrm{e}^{-\beta T}}{\beta} = 0, \end{aligned} \tag{7}$$

whereas the optimal reverse promotion (i.e., $p_2 > p_1$) and duration are the solution of

$$\begin{aligned} &-[\pi_{no\text{-}ref}(p_1) - \pi_{no\text{-}ref}(p_2)] - [\gamma_l(p_2 - p_1) \\ &\quad - (\gamma_g - \gamma_l)(p_1 - c)](p_2 - p_1)\mathrm{e}^{-\beta T} = 0, \\ &\pi'_{no\text{-}ref}(p_2)T + [(\gamma_g - \gamma_l)(p_1 - c) - 2\gamma_l(p_2 - p_1)] \\ &\quad \times \frac{1 - \mathrm{e}^{-\beta T}}{\beta} = 0. \end{aligned} \tag{8}$$

In general, one cannot solve systems (7) or (8) explicitly. An explicit solution is possible, however, when the demand function is linear, i.e., when

$$Q_{no\text{-}ref} = a - \delta p. \tag{9}$$

In this case, $\pi_{no\text{-}ref} = (p - c)(a - \delta p)$ and the optimal price in the absence of reference-price effects is

$$p_1 = \frac{a + \delta c}{2\delta}. \tag{10}$$

Proposition 8:
Let (1)–(5) hold, let $\gamma_l < \gamma_g$, let the demand function be given by (9), and let p_1 be the optimal price (10). Then, the optimal durations of a promotion and a reverse promotion are given by $T^{\mathrm{opt}} = x_0/\beta$, where x_0 is the positive root of the nonlinear equation

$$1 - \mathrm{e}^{-x} - 2x\mathrm{e}^{-x} - S\mathrm{e}^{-x}(1 - \mathrm{e}^{-x}) = 0, \tag{11}$$

and S is a parameter whose value is given by

$$S = \begin{cases} \dfrac{\gamma_g}{\delta} & \text{(promotion)}, \\ \dfrac{\gamma_l}{\delta} & \text{(reverse-promotion)}. \end{cases} \tag{12}$$

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The optimal promotion price is given by

$$p_2^{\text{opt}} = \begin{cases} p_1 - \dfrac{(\gamma_g - \gamma_l)(p_1 - c)}{\delta} g(x_0) & \text{(promotion)}, \\ p_1 + \dfrac{(\gamma_g - \gamma_l)(p_1 - c)}{\delta} g(x_0) & \text{(reverse-promotion)}, \end{cases} \quad (13)$$

where $g(x) = (1 - e^{-x})e^{-x}/2(1 - e^{-x} - xe^{-x})$. The additional profits resulting from the optimal promotion are given by

$$\Delta\Pi[p_2^{\text{opt}}, T^{\text{opt}}] = \pi_{\textit{no-ref}}(p_1)\left(\frac{\gamma_g - \gamma_l}{\delta}\right)^2 \times \frac{g(x_0)(1 - e^{-x_0})}{2\beta}. \quad (14)$$

Proof:
See Appendix.

The parameter S can be viewed as the ratio of the strength of reference-price effects to that of price effects. Since there are no explicit expressions for x_0 and $g(x_0)$ as a function of S, in Figure 1 we plot x_0, $g(x_0)$ and $g(x_0)(1 - e^{-x_0})$ as a function of S, which we calculated by solving Equation (11) numerically.

The above calculations lead to the surprising result that the optimal reverse promotion is more profitable than the optimal promotion.

Proposition 9:
Let (1)–(5) hold, let $\gamma_l < \gamma_g$ and let p_1 be the monopolistic optimal price (6). Then the optimal reverse promotion is more profitable than the optimal promotion.

Proof:
See Appendix.

The last result is somewhat surprising, since almost all price promotions involve a price reduction and not a price increase. Of course, one has to bear in mind that this result is only valid when $\gamma_l < \gamma_g$, and that the relation $\gamma_l < \gamma_g$ was found in a single aggregate level analysis (Greenleaf, 1995) and in only few disaggregate level investigations (e.g., Murthi and Kalyanaram, 1999). Most disaggregate empirical studies, however, suggest that $\gamma_l \geqslant \gamma_g$ (e.g., Winer, 1986; Lattin and Bucklin, 1989). In that case, profits under

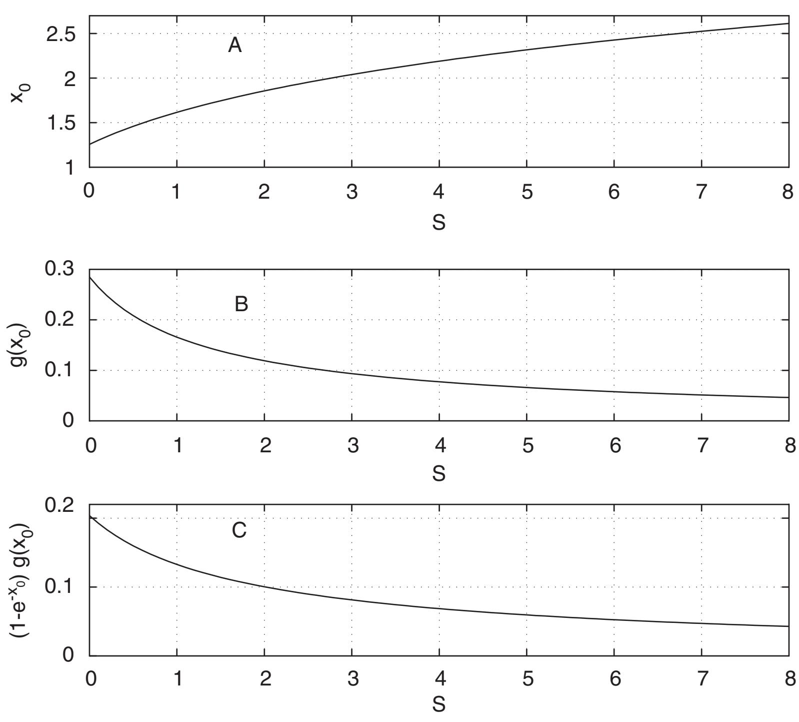

Figure 1. x_0, $g(x_0)$, and $g(x_0)(1 - e^{-x_0})$ as a function of S, which are used in the calculation of T^{opt}, p_2^{opt}, and $\Delta\Pi[p_2^{\text{opt}}, T^{\text{opt}}]$, respectively.

reverse promotions are lower than under promotions, as is evident from Proposition 4.

MULTIPLE PROMOTIONS

In the analysis thus far we have considered the case of a single price promotion. When analyzing the profitability of multiple price promotions, the key parameter is the time between the end of one promotion and the beginning of the next promotion. If this time is sufficient in length (i.e. $\gg 1/\beta$), then by the time the next promotion begins, the effect of the previous promotion on reference price has already disappeared. In that case, each price promotion is 'independent' from others, and overall profitability is simply the sum of the profitability of all price promotions.

When price promotions are more frequent, reference price at the beginning of a promotion is below p_1, as consumers do not have enough price exposures to 'forget' the previous promotion price. In that case, the expressions for the profitability of each promotion are different from those calculated earlier, since in those calculations we assumed that $r = p_1$ at the beginning of the price promotion (Equation (4)). Of course, one can still calculate explicitly the profitability of the promotions as we did in the Appendix. In fact, the only difference would be that the resulting expressions would be slightly more complex.

An interesting question in the context of multiple promotions is what is the optimal interpromotion time. This question is of interest only in the case where gain effects are stronger than loss effects, as in the loss-aversive case promotions are not profitable. Let us first consider the case of multiple promotions with the optimal duration T and promotion price p_2 that were calculated earlier. Clearly, in order to utilize reference-price effects, a sufficient period of time should elapse between the promotions, so that consumers 'forget' the previous promotion price. In contrast, no benefit is derived from having the promotions too wide apart from each other, as the profitability of each promotion is then independent of the interpromotion time but the frequency of profitable promotions is small. Therefore, there is an optimal interpromotion time which is, of course, of the order of $1/\beta$. Note, however, that there is no reason why retailers should limit themselves to a policy of repeating the same price promotion at fixed intervals, nor is there a reason why the values of T and p_2 which are optimal for a single price promotion would remain optimal in the multiple promotions case. We are thus led to a more general question, namely, what is the optimal pricing policy over a finite or infinite planning horizon. This issue is further discussed in a later section.

PRICE RIGIDITY

The issue of price rigidity or flexibility plays an important role in marketing and economics theory. Consider, for example, a firm facing a cost shock. As a result, the 'old' price of the product is not optimal any more. Nevertheless, firms do not always change the price to the new optimal level. Recent micro-level price rigidity analysis reveals price rigidity variations between a manufacturer and a retailer depending on the cost shock (Levy *et al.*, 2002). We now expand on the micro-level price rigidity literature by incorporating consumer evaluations of price changes (i.e., reference-price effects). Indeed, since we showed that when gain effects are larger than loss effects it is in the firm's interest to change its price even without a price shock (Proposition 7), in that case reference-price effects contribute to price flexibility. In contrast, when consumers are loss aversive, prices would tend to be more rigid because of the negative effect of reference price on profits (Proposition 5).

While the above statements are qualitative, we can also *quantify* the effect of reference price on price rigidity. To illustrate, let us consider the case of loss-aversive buyers and a firm that at time t_0 faces a temporary cost shock of duration T, during which its cost per unit increases from c to $c + \Delta c$. For simplicity we assume a linear demand function $Q = a - \delta p$. Therefore, the optimal price is $p_1 = (a + \delta c)/2\delta$ before the cost change and $p_2 = (a + \delta(c + \Delta c))/2\delta$ during the cost change. Let us compare profits under strategy I where the firm does not change its price during the cost change, and strategy II where the firm raises its price to p_2 for the duration of the cost change. From Proposition 4 the loss of profits under strategy II due to reference-price effects is given by

$$\Delta\Pi_{ref} = -e^{-\alpha t_0}\frac{1 - e^{-\beta T}}{\beta}\left[(\gamma_l - \gamma_g)\frac{a - \delta c}{2\delta} + \gamma_l\frac{\Delta c}{2}\right]\frac{\Delta c}{2}.$$

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The increase in profits under strategy II due to the change to the optimal price p_2 is given by

$$\Delta\Pi_{no\text{-}ref} = \int_{t_0}^{t_0+T} e^{-\alpha t}[(a - \delta p_2)(p_2 - (c + \Delta c)) - (a - \delta p_1)(p_1 - (c + \Delta c))]\, dt \approx e^{-\alpha t_0} T\delta(\Delta c)^2.$$

The condition for price rigidity is $\Delta\Pi_{no\text{-}ref} + \Delta\Pi_{ref} < 0$. It thus follows that when $T\delta < \gamma_l(1 - e^{-\beta T})/4\beta$, it is optimal for the firm to stay at p_1 regardless of the shock change. When $T\delta > \gamma_l(1 - e^{-\beta T})/4\beta$, it is optimal for the firm to stay at p_1 provided that

$$\Delta c < \frac{(1 - e^{-\beta T})(\gamma_l - \gamma_g)(a - \delta c)/\delta}{4\beta T\delta - (1 - e^{-\beta T})\gamma_l}.$$

Therefore, reference-price effects provide another explanation of why prices may be rigid for small shocks.

FINAL REMARKS

In this study we have developed a methodology for calculating the profits of a price promotion in the presence of asymmetric reference-price effects and for calculating the optimal depth and duration of the promotion. Our model can be easily extended to more complex situations, such as multiple promotions, consumer heterogeneity and competition (see Appendix), and other reference-price processes such as reference brand framework (Hardie *et al.*, 1993). Such additions do not affect the model conclusions, e.g., that price promotions are only profitable when effects of gains are larger than losses. As in any quantitative model in marketing, however, we cannot include all possible variables influencing price promotion. Thus, in some cases a retailer may need to add additional elements to the model. Nevertheless, this study can still be relevant in that case, since (1) these additions can still be carried out using the methodology developed in this study, and (2) our calculation of the reference-price component on the overall profitability of a price promotion would still be valid.

As we discussed earlier, our results are related to the problem of finding the optimal pricing strategy in the presence of reference-price effects. Greenleaf (1995) and Kopalle *et al.* (1996) calculated numerically the optimal pricing strategies using dynamic programming. These simulations showed that the optimal pricing strategy is constant in the loss-aversive case, but is cyclical ('chattering') in the 'gain-aversive' case. In Kopalle *et al.* (1996) these observations were also shown analytically by a long, complex proof. Both the numerical and the analytical approaches, however, did not provide a simple, intuitive explanation as to why the optimal pricing strategy is cyclical in one case, yet constant in the other. The initial motivation for this study was to do just that. Indeed, by focusing on the profitability of a single price promotion and isolating the impact of reference price effects on profitability we were able to show that in the loss-aversive case reference-price effects always lead to a decline in the profitability of a promotion, whereas in the 'gain-aversive' case reference-price effects can increase the profitability of a promotion. Thus, in the loss-aversive case the best strategy is simply to set a constant price. This is, however, not the optimal strategy in the 'gain-aversive' case, since, as we have seen, promotions can increase profits. Clearly, the more promotions utilized, the higher the profits, thus explaining why in that case the optimal pricing strategy is cyclical. It is perhaps worth noting that the analytical results of Kopalle *et al.* (1996) provide qualitative results on the optimal pricing strategies (i.e., cyclical or constant), but do not give the optimal depth and duration of a promotion, for which we obtain explicit expressions.

One reason why retailers might not adopt the optimal pricing strategies which are mentioned above is that they can involve frequent price changes, which result in additional costs which were not considered here. Indeed, it was shown theoretically (Mankiw, 1985) and empirically (Levy *et al.*, 1997, 1998; Dutta *et al.*, 1999) that one reason for price rigidity is the high cost of price adjustment ('menu costs'). Therefore, retailers frequently prefer to have a constant price with infrequent promotions. In such a case, our explicit expressions of the effect of reference price on profits, and of the optimal depth and duration of promotion, can be useful.

APPENDIX

Available electronically at www.math.tau.ac.il/~fibich, www.bgu.ac.il/~ariehg

NOTE

1. The continuous memory parameter β is roughly proportional to 1/(average time between exposures of a consumer to the product price). Therefore β is of the order of 1/(several weeks) for frequently purchased goods, whereas α is of the order of 1/year.

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