Calculus A for Economics

Exercise Number 1

1) Define the functions $f(x) = \frac{x-2}{x+1}$ and $g(x) = \frac{|x-2|}{x+1}$. Compute:
a) $f(0)$; $f(\sqrt{2})$; $g(4)$ $b) f(t+1) - f(t)$

2) For each of the following functions find the domain of definition:

$$a)\ \ y = \sqrt{5-2x} \qquad b)\ \ y = \frac{1}{\sqrt{|x|-x}}$$

$$c)\ \ y = \frac{x}{\sqrt{x^2-3x+2}} \qquad d)\ \ y = \sqrt{\frac{x-2}{x+2}} + \sqrt{\frac{1-x}{1+x}}$$

3) Prove that $||a| - |b|| \leq |a-b|$ for all $a, b \in \mathbf{R}$.
(*Hint:* Compute $||a| - |b||^2$).

4 Determine which of the following functions is one to one:

$$a)\ \ f(t) = \sqrt{1-t} \qquad b)\ \ g(x) = x + \frac{1}{x}$$

5) Determine which of the following functions are even, which are odd, and which are not even and not odd:

$$a)\ \ y = x - x^2 \qquad b)\ \ y = x - x^3 + x^5 \qquad c)\ \ y = \frac{a^x + a^{-x}}{2}$$

6) Show that for any function $f(x)$, the function $f(x) + f(-x)$ is even.

7) Find the range of the following functions:

$$a)\ \ F(x) = \sqrt{1-x^2} \qquad b)\ \ h(x) = \frac{1}{\sqrt{1-x}}$$

8) For $f(x) = x + 1$ and $g(x) = x - 2$, solve the equation $|f(x) + g(x)| = |f(x)| + |g(x)|$.

9) Give a sketch of the following functions:

$$a)\ \ y = 2x + 3 \qquad b)\ \ y = x^2 - 5x + 6$$
$$c)\ \ y = \sqrt{x-1} \qquad d)\ \ y = x^3$$
$$e)\ \ y = -|x-2| \qquad f)\ \ y = |x| - x$$

10) Given the function $f(x) = ax^2+bx+5$, find a and b such that $f(x+1)-f(x) = 8x+3$ for all x.

11) Write the function $f(x) = x^2 + 3x + 2$ as a sum of an even function and an odd function.

12) Let $f(x) = [x]$ where $[x]$ denotes the first integer number which is less or equal to x. For example $[2.3] = 2$; $[\frac{1}{3}] = 0$; $[4] = 4$; $[-2.9] = -3$. Sketch the graph of the following functions:

a) $f(x) = [x]$.

b) $f(x) = x - [x]$.