Calculus A for Economics

Exercise Number 10

1) Find *all* asymptotes of the following functions:

$a)\ \ y = \dfrac{1}{x}$ $\qquad b)\ \ y = \dfrac{1}{x^2 - 4x + 5}$ $\qquad c)\ \ \dfrac{x^3}{2(x+1)^2}$

$d)\ \ y = e^{-x^2}$ $\qquad e)\ \ y = x\ln x$ $\qquad f)\ \ y = xe^x$

2) Let $f(x)$ be a rational function.

a) Prove that if the limit $\lim_{x\to\infty} \frac{f(x)}{x}$ exists and is equal to a, then $y = ax+b$ is an asymptote for $f(x)$ at infinity. Here $b = \lim_{x\to\infty}[f(x) - ax]$.

b) Prove that if $y = ax + b$ is an asymptote for $f(x)$ at infinity, then it is also an asymptote to $f(x)$ at minus infinity.

3) Study the following functions according to the following points:
domain of definition, points of intersection with the axes, domains of continuity, extreme points, domains of increasing and decreasing, point of inflexion, domains of concavities, all asymptotes.
With this data give a graphic description of the function.

$a)\ \ y = \dfrac{x}{3x-1}$ $\qquad b)\ \ y = \dfrac{x+3}{x-2}$ $\qquad c)\ \ y = 1 + (x-2)^{5/3}$

$d)\ \ y = \sqrt{x - x^2}$ $\qquad e)\ \ y = \dfrac{1}{1+x^2}$ $\qquad f)\ \ y = \dfrac{2x-1}{(x-1)^2}$

$g)\ \ y = \dfrac{x^3}{3-x^2}$ $\qquad h)\ \ y = \dfrac{x^4}{x^3-1}$ $\qquad i)\ \ y = xe^{-x}$

$j)\ \ y = x - \ln(x+1)$ $\qquad k)\ \ y = \ln(x^2+1)$ $\qquad l)\ \ y = x^2e^{-x^2}$

$m)\ \ y = \dfrac{\ln x}{x}$ $\qquad n)\ \ y = x^3e^{-x}$