Calculus A for Economics

Exercise Number 2

1) Compute both $f \circ g$ and $g \circ f$ for the following functions:

$a)$ $f(x) = 2x + 5; \quad g(x) = x^2$ $\qquad\qquad$ $b)$ $f(x) = \dfrac{1}{x}; \quad g(x) = \dfrac{1}{x}$

$c)$ $f(x) = e^{x+1}; \quad g(x) = \ln x$

2) Compute $f \circ g \circ h$ for the following functions:

$a)$ $f(x) = 4x - 8; \quad g(x) = \dfrac{x}{4}; \quad h(x) = \sqrt{x}$

$b)$ $f(x) = \dfrac{1}{x}; \quad g(x) = e^x; \quad h(x) = \sqrt{x}$

$c)$ $f(x) = \ln x; \quad g(x) = x^2 + 3; \quad h(x) = \dfrac{1}{x}$

3) For the following functions, give a formal formula for $f^{-1}(x)$. Determine the domain of definition of $f(x)$ and $f^{-1}(x)$, and deduce for which values of x these functions are inverse to one another.

$a)$ $f(x) = x^2 + 1$ $\qquad\quad$ $b)$ $f(x) = \sqrt[3]{x^2 + 1}$ $\qquad\qquad$ $c)$ $f(x) = \dfrac{2x + 3}{x - 1}$

$d)$ $f(x) = 10^{x+1}$ $\qquad\qquad$ $e)$ $f(x) = 1 + \ln(x + 2)$ $\qquad\quad$ $f)$ $f(x) = \dfrac{2^x}{1 + 2^x}$

4) For $x > 0$ let $f(x) = \sqrt[n]{a - x^n}$. Show that $f(f(x)) = x$. Find $f^{-1}(x)$.

5) Find the domain of definition of the following functions:

$a)$ $y = \ln \dfrac{x}{4}$ $\qquad$ $b)$ $y = \log_{10} \dfrac{1 - 2x}{4}$ $\qquad$ $c)$ $y = \dfrac{3}{4 - x^2} + \log_{10}(x^3 - x)$

6) Given the graph of the function $f(x)$, give a sketch of the function $f^{-1}(x)$. (See the file graph2)

7) Give a sketch of the function

$$f(x) = \begin{cases} 2x + 3, & x \leq 0 \\ x^2 - 1, & 0 < x \leq 3 \\ \frac{1}{x}, & x > 3 \end{cases}$$

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8) For what values of a, b, c and d the function $f(x) = \frac{ax+b}{cx+d}$ satisfies $(f \circ f)(x) = x$.

9) Let $f(x) = x + 1$. Find a function $g(x)$ such that $(f \circ g)(x) = (g \circ f)(x)$.

10) Find $f^{-1}(x)$ for the function

$$f(x) = \begin{cases} -x^2, & x \geq 0 \\ 1 - x^3, & x < 0 \end{cases}$$