Calculus A for Economics

Exercise Number 3

1) Using *only* the definition of a limit of a function, prove the following:

$a)\ \lim_{x\to 2}(3x-1)=5 \qquad b)\ \lim_{x\to 3}(6x-7)=11 \qquad c)\ \lim_{x\to 2}x^2=4$

2) It is given that $\lim_{x\to c} f(x)=2,\ \lim_{x\to c} g(x)=-1$ and $\lim_{x\to c} h(x)=0$. Compute:

$$a)\ \lim_{x\to c}[f(x)]^2 \qquad b)\ \lim_{x\to c}\frac{h(x)}{f(x)} \qquad c)\ \lim_{x\to c}\frac{1}{f(x)-g(x)}$$

3) It is given that $\lim_{x\to c} g(x)=0$ and that $f(x)g(x)=1$ for all values of x. Prove that $\lim_{x\to c} f(x)$ does not exist.

4) Compute the following limits:

$$a)\ \lim_{x\to 2}\frac{x^2+x+1}{x^2+2x} \qquad b)\ \lim_{x\to 2}\frac{x-2}{x^2-4} \qquad c)\ \lim_{x\to 1}\frac{x^2-2x+1}{x^3-x}$$

$$d)\ \lim_{x\to 0}\frac{\sqrt{1+x}-1}{x} \qquad e)\ \lim_{x\to 5}\frac{\sqrt{x-1}-2}{x-5}$$

5) It is given that $\lim_{x\to c} f(x)=l$ where $l\neq 0$ and that $\lim_{x\to c} g(x)=0$. Prove that $\lim_{x\to c}\frac{f(x)}{g(x)}$ does not exist.
Hint: use the identity $f(x)=g(x)\frac{f(x)}{g(x)}$.

6) Use exercise **5)** to prove that the limit $\lim_{x\to 1}\frac{x}{x^2-1}$ does not exist.

7) Use the definition of the limit of a function to prove the following:

$$a)\ \lim_{x\to 2}x^4=16 \qquad b)\ \lim_{x\to 2}\frac{1}{x}=\frac{1}{2}$$

8) Give an example of two functions $f(x)$ and $g(x)$ such that both limits $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist, but the limit $\lim_{x\to 0}(f(x)+g(x))$ do exist.

9) Assume that for the two given functions $f(x)$ and $g(x)$, the limit $\lim_{x\to 0} f(x)$ exist but $\lim_{x\to 0} g(x)$ does not exist. Prove that $\lim_{x\to 0}(f(x)+g(x))$ does not exist.