Calculus A for Economics

Exercise Number 4

1) Let $f(x)$ be a function which satisfies $-|x| \leq f(x) \leq |x|$ for all $x \neq 0$. Compute $\lim_{x\to 0} f(x)$.

2) If $|\frac{f(x)}{x}| \leq 1$ for all $x \neq 0$, compute $\lim_{x\to 0} f(x)$.

3) For all real numbers $x, c > 0$ prove the following inequality

$$0 \leq |\sqrt{x} - \sqrt{c}| = \frac{|x-c|}{\sqrt{x}+\sqrt{c}} \leq \frac{1}{\sqrt{c}}|x-c|$$

Use it to prove that $\lim_{x\to c} \sqrt{x} = \sqrt{c}$.

4) Given the function:

$$g(x) = \begin{cases} x^2, & x < 1 \\ x, & 1 < x < 4 \\ 4 - x, & x > 4 \end{cases}$$

compute the following limits if they exist:

a) $\lim_{x\to 1^-} g(x)$ *b)* $\lim_{x\to 2^-} g(x)$ *c)* $\lim_{x\to 4} g(x)$

5) Compute the following one side limits:

a) $\lim_{x\to 2^+} \frac{x^2}{x+2}$ *b)* $\lim_{x\to 0^-} \frac{1+\sqrt{x}}{1-\sqrt{x}}$ *c)* $\lim_{x\to 1^-} \sqrt{|x| - x}$

6) Compute the following limits:

a) $\lim_{x\to\infty} (\sqrt{x+a} - \sqrt{x}) \quad a \neq 0$

b) $\lim_{x\to\infty} (\sqrt{x^2+1} - x)$

c) $\lim_{x\to -\infty} (\sqrt{x^2+1} - x)$

d) $\lim_{x\to\infty} (\sqrt{x^2-2x-1} - \sqrt{x^2-7x+3})$

e) $\lim_{x\to\infty} \frac{1+x-3x^3}{1+x^2+3x^3}$

f) $\lim_{x\to\infty} \left(\frac{x^3}{x^2+1} - x\right)$

g) $\lim_{x\to 0} \frac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4}$

h) $\lim_{x\to\infty} \frac{x^4-5x}{x^2-3x+1}$

i) $\lim_{x\to 1} \frac{(x-1)\sqrt{2-x}}{x^2-1}$

j) $\lim_{x\to\infty} \frac{a^x}{a^x+1} \quad (0 < a \neq 1)$

k) $\lim_{x\to\infty} \frac{a^x - a^{-x}}{a^x + a^{-x}} \quad (0 < a \neq 1)$

7) Given that $-1 \leq f(x) \leq 1$, use the Sandwich Theorem to prove that $\lim_{x\to\infty} |\frac{f(x)}{x}| = 0$.