# Calculus A for Economics

## Exercise Number 6

**1)** Use the definition of the derivative in order to compute the derivatives of the following functions at the point $x_0$:

$a)\ \ f(x) = 3x + 5; \quad x_0 = -1 \qquad\qquad b)\ \ f(x) = (x-1)^2; \quad \text{any} \quad x_0$

$c)\ \ f(x) = \dfrac{1}{\sqrt{x-1}}; \quad \text{any} \quad x_0 > 1$

**2)** For the following functions find the points where they are differentiable, and compute the derivative at these points

$a)\ \ f(x) = |2x - 5| \qquad\qquad b)\ \ f(x) = \sqrt{x+1}$

**3)** Assume that $f(x)$ and $g(x)$ are both differentiable at zero and that $f(0) = g(0) = 0$. Prove that if $g'(0) \neq 0$, then

$$\lim_{x\to 0} \frac{f(x)}{g(x)} = \frac{f'(0)}{g'(0)}$$

**4)** Assume that $f(x)$ is differentiable at $x = a$. Prove that

$$\lim_{x\to 0} \frac{(x+a)f(a) - af(x+a)}{x} = f(a) - af'(a)$$

**5)** Prove that the derivative of an even function is an odd function.

**6)** Differentiate the following functions:

$a)\ \ y = 3x^5 - x^3 + \dfrac{1}{2} \qquad b)\ \ y = (x^2+1)(x^5-3) \qquad c)\ \ y = (\sqrt{x}+1)(\dfrac{1}{\sqrt{x}} - 1)$

$d)\ \ y = x^2 - \dfrac{1}{x^{1/2}} + \dfrac{\sqrt[3]{x^2}}{3x} \qquad e)\ \ y = \dfrac{x^3 - 2x}{x^2+x+1} \qquad f)\ \ y = \dfrac{1}{x^2+x+1}$

$g)\ \ y = \dfrac{ax + bx^2}{am + bm^2} \quad a, b, m \in \mathbf{R} \qquad h)\ \ y = \dfrac{3-x}{(1-x^2)(1-2x^3)}$

**7)** Find the equations of the tangent line and the normal line to the function $y = x + \frac{1}{x}$ at the point $x = 5$.

**8)** Find a point on $y = (x-1)^3$ such that the tangent line at that point is parallel to the line $y = x$.

**9)** Find points on the function $y = \frac{1}{x}$ on which the tangent line is perpendicular to the line $y = x$.

**10)** Find numbers $A, B, C$ such that $y = Ax^2 + Bx + C$ will pass through the point $(1, 3)$ and at the point $(2, 0)$ its tangent line is $y = 8 - 4x$.

**11)** For $F(x) = (x-1)(x-2)(x-3)$ compute $F'(1)$.

**12)** Differentiate $y = (1+\sqrt{x})(1+\sqrt{2x})(1+\sqrt{3x})$.

**13)** Given $g(x) = \frac{a-x}{1+x}$ compute $g'(1)$.

**14)** Determine the values of $a$ and $b$ such that the two functions $y = x^2 - 2$ and $y = 2x^2 + 2ax + b$ will have the same tangent line.

**15)** Assume that $f(x)$ is defined for all $x$, and satisfies the inequality $|f(x)| \leq x^2$ for all $x$. Prove that $f'(0) = 0$.

**16)** Assume that $f(x) = xg(x)$ and that $g(x)$ is continuous at $x = 0$. Prove that $f'(0)$ exists and express it in terms of values of $g(x)$.