

Calculus A for Economics

Exercise Number 7

1) Differentiate the following functions:

a) $y = \frac{e^x + e^{-x}}{3}$

b) $y = \left(x + \frac{1}{x}\right)^3$

c) $y = \left(\frac{ax + b}{cx + d}\right)^3$

d) $y = \left(\frac{x^3}{3} + \frac{x^2}{2} + x\right)^{-1}$

e) $y = \sqrt{\frac{x+1}{x-1}}$

f) $y = (\ln x)^2$

g) $y = \log_3(x^2 - 1)$

h) $y = \sqrt{1 + e^x}$

i) $y = \frac{1 - 10^x}{1 + 10^x}$

j) $y = x^2 e^{-\frac{x^2}{a^2}}$

2) Given that $g(0) = 2$; $g'(0) = 1$ and $f'(2) = 5$ compute $(f \circ g)'(0)$.

3) Compute

$$\frac{d}{dx} \left[f \left(\frac{x-1}{x+1} \right) \right]$$

4) Given that $x = y^3 - 4y + 1$ compute $\frac{dy}{dx}$.

5) Let $y = \ln(x^2 - 1)$. Verify that $\frac{dy}{dx} \frac{dx}{dy} = 1$.

6) a) For $f(x) = \frac{x}{1+x}$ compute $f''(x)$.

b) For $y = ax^4$ compute $\frac{d^4y}{dx^4}$.

c) Compute $\frac{d^2}{dx^2} \left(x^3 + \frac{1}{x^3} \right)$.

7) Prove that $y = \sqrt{2x - x^2}$ satisfies $y^3 y'' + 1 = 0$.

8) Differentiate the following functions:

a) $y = x^{(x+2)}$

b) $y = \frac{(x+2)(x^2+1)(x-3)}{(3x+1)(x-5)^2}$

c) $y = (\ln x)^x$

d) $y = \left(\frac{x}{1+x} \right)^x$

9) Compute the following limits:

a) $\lim_{x \rightarrow a} \frac{x-a}{x^n - a^n}$

b) $\lim_{x \rightarrow 0} \frac{1+x-e^x}{x(e^x-1)}$

c) $\lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}}$

d) $\lim_{x \rightarrow 1} \frac{\sqrt[n]{x} - 1}{\sqrt[n]{x} - 1}$

e) $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x} \right)^x$

f) $\lim_{x \rightarrow 0} (e^x + x)^{\frac{1}{x}}$

g) $\lim_{x \rightarrow 1} x^{\frac{1}{x-1}}$

h) $\lim_{x \rightarrow 1} \frac{x^{1/2} - x^{1/4}}{x-1}$

10) Are the functions $(f \circ f)'(x)$ and $f'(f(x))$ always equal?

11) Consider the function

$$f(x) = \begin{cases} \frac{g(x)}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

where $g(x)$ is differentiable twice near zero. Also assume that $g''(x)$ is continuous at $x = 0$.

It is given that $g(0) = g'(0) = 0$ and that $g''(0) = 17$. Compute $f'(0)$.