Calculus A for Economics

Exercise Number 9

1) For the following functions find all extreme points:

$a)\ y = x + \frac{1}{x}$ $\quad b)\ y = x^2(1-x)$ $\quad c)\ y = |x^2 - 16|$

$d)\ y = -\frac{x^3}{x+1}$ $\quad e)\ y = x - \ln(1+x)$

$f)\ y = ae^{px} + be^{-px} \quad (a, b, p > 0)$ $\quad g)\ y = xe^{-x}$

$h)\ y = xe^{x^2}$ $\quad i)\ y = \ln\frac{x}{1+x^2}$ $\quad j)\ y = \ln\frac{x^3}{x-1}$

2) For each of the functions in exercise **1)** find the domains of increasing and decreasing.

3) For each of the following functions find all the inflection points and where the functions are concave up and where they concave down:

$a)\ y = (1-x)^2(1+x)^2$ $\quad b)\ y = (x-3)^{1/5}$ $\quad c)\ y = \frac{1-\sqrt{x}}{1+\sqrt{x}}$

$d)\ y = x - \ln x$ $\quad e)\ y = x^2 e^{-x}$

$f)\ y = \frac{a}{x}\ln\frac{x}{a} \quad (a > 0)$ $\quad g)\ y = e^{-x^2}$

4) For each of the following functions find the extreme points in the given intervals:

$a)\ y = x + 2\sqrt{x} \quad \text{in} \quad [0, 4]$ $\quad b)\ y = x^3 - 3x^2 + 6x - 2 \quad \text{in} \quad [-1, 1]$

$c)\ y = \frac{1-x+x^2}{1+x-x^2} \quad \text{in} \quad [0, 1]$ $\quad d)\ y = e^{-x^2} \quad \text{in} \quad [0, 3]$

$e)\ y = \ln x \quad \text{in} \quad [1, e]$

5) Let $f(x)$ and $g(x)$ be two functions which are differentiable in (a, b) and continuous in $[a, b]$. Assume that $f(a) = g(a)$ and that $f'(x) > g'(x)$ for all $x \in (a, b)$. Prove that $f(x) > g(x)$ for all $x \in (a, b)$.

6) Use exercise **5)** to prove the following inequalities. In d) use induction on n and differentiation:

$a)\ 2\sqrt{x} > 3 - \frac{1}{x} \quad (x > 1)$ $\quad b)\ e^x > 1 + x \quad (x \neq 0)$

$c)\ \ln x > \frac{2(x-1)}{x+1} \quad (x > 1)$ $\quad d)\ 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} < e^x \quad (x > 0)$

7) Find two positive numbers whose sum is 40 such that their product is maximal.

8) Find the sizes of a box whose bases are rectangles, such that its volume is maximal, if it is given that the area of its faces is 200 square meters, and that the length of its base should be three times its width.

9) A factory can manufactor up to 25 products a week. It is known that if n products are manufactored every week, and are sold at the price of $110-2n$ Shekel each product, then the cost of manufactoring n products is $600+10n+n^2$ Shekel. How many products should be sold a week in order to guarantee a maximal profit.

10) A window has a shape of a rectangle and a half a circle above it. One has to surround it with a rope whose length is p meters. Find the radius of the half circle such that the area of the window will be maximal.

11) We are given a rectangle whose sides are of length a and b. From each corner of the rectangle we cut a small square of the same size. Then we fold the shape to obtain a box whose upper part is open. What should be the size of the squares so that the obtained box will be of maximal volume.

12) Among all the right triangulares located in the first quadrant, whose edges are the two axes and the straight line which passes through the point $(1,2)$, find the one which has minimal area. (A right triangular is a triangular whose one of his angles is 90 degrees).

13) Write the number 36 as a product of two numbers such that the sum of their squares will be minimal.

14) Let $a>0$. Prove that the maximum value that the function

$$f(x)=\frac{1}{1+|x|}+\frac{1}{1+|x-a|}$$

obtains is $\frac{2+a}{1+a}$.