TRANSLATION-FINITE SETS

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Abstract. The families of right (left) translation finite subsets of a discrete infinite group Γ are defined and shown to be ideals. Their kernels Z_R and Z_L are identified as the closure of the set of products pq $(p \cdot q)$ in the Čech-Stone compactification $\beta\Gamma$. Consequently it is shown that the map $\pi : \beta\Gamma \to \Gamma^{WAP}$, the canonical semigroup homomorphism from $\beta\Gamma$ onto Γ^{WAP}, the universal semitopological semigroup compactification of Γ, is a homeomorphism on the complement of $Z_R \cup Z_L$.

Introduction

This note is an elaboration on the beautiful work of Ruppert [3] from 1985. Given a discrete infinite group Γ we define right and left versions of the combinatorial property (of subsets of Γ) of being translation finite. Then, using the ultrafilter representation of the Čech-Stone compactification $\beta\Gamma$, we show that the collections of sets with these properties form ideals (Theorem 2.3). This yields a new proof of Ruppert's theorem which asserts that the collection of translation finite sets forms an ideal. We then use these results to obtain some unexpected information about the map $\pi : \beta\Gamma \to \Gamma^{WAP}$, the canonical semigroup homomorphism from $\beta\Gamma$ onto Γ^{WAP}, the universal semitopological semigroup compactification of Γ (Theorem 2.4).

1. The C^*-algebras $\ell_\infty(\Gamma)$ and $WAP(\Gamma)$

Let Γ be a countable discrete infinite group with unit element e. We briefly review some basic properties of the C^*-algebras $\ell_\infty(\Gamma)$, of bounded complex-valued functions on Γ, and $WAP(\Gamma)$, the closed subalgebra comprising the weakly almost periodic functions on Γ. Recall that $f \in \ell_\infty(\Gamma)$ is *weakly almost periodic* if its orbit under translations $\{f \circ \gamma : \gamma \in \Gamma\}$ is a weakly precompact subset of the Banach space $\ell_\infty(\Gamma)$. We are mostly interested in their Gelfand (or maximal ideal) spaces: $\beta\Gamma$, the Čech-Stone compactification of Γ, and Γ^{WAP}, the universal WAP-compactification of Γ, respectively.

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The compactification $\beta\Gamma$ can be viewed as the collection of ultrafilters on Γ, where an element $\gamma \in \Gamma$ is presented as the principal ultrafilter $e_\gamma = \{A \subset \Gamma : \gamma \in A\}$. Then the left translation of an ultrafilter $q \in \beta\Gamma$ by γ is the ultrafilter $\gamma q = \{A \subset \Gamma : \gamma^{-1}A \in q\}$ (note that this extends the product on Γ as $\gamma e_\delta = e_{\gamma\delta}$) . These translations define a left action of Γ on $\beta\Gamma$ and the resulting pointed dynamical system $(\beta\Gamma, e, \Gamma)$ is the *universal ambit* (or point transitive pointed system). That is, for any point transitive pointed Γ dynamical system (Y, y_0, Γ) there is a unique homomorphism of pointed dynamical systems $\pi : (\beta\Gamma, e, \Gamma) \to (Y, y_0, \Gamma)$.

This Γ action on $\beta\Gamma$ can be extended to a multiplication on $\beta\Gamma$ as follows: for $p, q \in \beta\Gamma$

$$m_R(p, q) = pq = \{A \subset \Gamma : \{\alpha \in \Gamma : \alpha^{-1}A \in q\} \in p\}.$$

This multiplication has the property that for each fixed $q \in \beta\Gamma$ the map $R_q : \beta\Gamma \to \beta\Gamma$, defined by $p \mapsto pq = m_R(p, p)$ is continuous. Thus this product makes $\beta\Gamma$ a *right topological semigroup.* It can be shown that this right topological semigroup can be identified with the enveloping semigroup $E(\beta\Gamma, \Gamma)$ of the dynamical system $(\beta\Gamma, \Gamma)$.

One can also define a left product on $\beta\Gamma$ by

$$m_L(p, q) = p \cdot q = \{A \subset \Gamma : \{\alpha \in \Gamma : A\alpha^{-1} \in p\} \in q\}.$$

This extension of the product on Γ to a product on $\beta\Gamma$ makes $\beta\Gamma$ a *left topological semigroup*, i.e. one in which the maps $L_q : \beta\Gamma \to \beta\Gamma$, defined by $p \mapsto q \cdot p = m_L(p, p)$, are continuous.

The *remainder space* of Γ is the compact space $X := \beta\Gamma^* = \beta\Gamma \setminus \Gamma$. Clearly X is a subsemigroup of $\beta\Gamma$ with respect to both right and left multiplications. We let $Z_R := \operatorname{cls} X^2 = \operatorname{cls}\{pq : p, q \in X\}$ and $Z_L := \operatorname{cls} X^{\cdot 2} = \operatorname{cls}\{p \cdot q : p, q \in X\}$. We also set $Z = Z_R \cup Z_L$.

As the algebra $C_0(\Gamma)$, comprising the functions on Γ which vanish at infinity, is contained in the algebra $WAP(\Gamma)$ we deduce that $WAP(\Gamma)$ distinguishes points in Γ and that consequently the natural compactification map of Γ into Γ^{WAP} is an isomorphism. We will therefore consider Γ as a dense discrete subset of both $\beta\Gamma$ and Γ^{WAP}.

A dynamical system (X, Γ) is called *weakly almost periodic* (WAP) if for every $F \in C(X)$, its orbit $\{F \circ \gamma : \gamma \in \Gamma\}$ forms a weakly precompact subset of the Banach space $C(X)$. A theorem of Ellis and Nerurkar which is based on well known results of Grothendiek asserts that a system (X, Γ) is WAP iff its enveloping semigroup $E(X)$ consists of continuous maps, iff $E(X)$ is a *semitopological semigroup* (that is, one in which both right and left multiplications are continuous). It then follows that the dynamical system Γ^{WAP} is the universal WAP point transitive dynamical system. Moreover, Γ^{WAP} is isomorphic to its own enveloping semigroup and is therefore also the maximal semitopological semigroup compactification of Γ.

Let $\pi : \beta\Gamma \to \Gamma^{WAP}$ denote the canonical homomorphism of the corresponding dynamical systems. With our identifications of Γ as a subset of both $\beta\Gamma$ and Γ^{WAP} we have $\pi(\gamma) = \gamma$ for every $\gamma \in \Gamma$. We set $Y := \Gamma^{WAP} \setminus \Gamma$. As a direct consequence of the discussion above we see that for every $p, q \in \beta\Gamma$ we have $\pi(pq) = \pi(p)\pi(q)$ and $\pi(p \cdot q) = \pi(p)\pi(q)$. Consequently $\pi(Z_R \cup Z_L) = \mathrm{cls}\, Y^2$. A result of our analysis shows that the restricted map

$$\pi : \beta\Gamma \setminus Z \to \Gamma^{WAP} \setminus \mathrm{cls}\, Y^2$$

is a homeomorphism (Theorem 2.4 below). This extends results of Ruppert and Hindman and Strauss (see [2, Theorem 21.22]).

2. Translation-finite sets

2.1. **Definitions.** 1. Let $Z_R = \mathrm{cls}\, X^2 \subset X$. Set

$$\mathcal{I}_R = \{A \subset \Gamma : \mathrm{cls}\, A \cap Z_R = \emptyset\}.$$

Set $\mathcal{F}_R = \{B \subset \Gamma : B^c \in \mathcal{I}\} = \{B \subset \Gamma : \mathrm{cls}\, B \supset Z_R\}$. Clearly $\mathcal{I}_R$ is an ideal and $\mathcal{F}_R$ is a filter.

2. Set $Z_L = \mathrm{cls}\, X^{\cdot 2} \subset X$, where $X^{\cdot 2} = \{p \cdot q : p, q \in X\}$. The ideal $\mathcal{I}_L$ and the filter $\mathcal{F}_L$ are then defined as above with Z_L replacing Z_R.
3. Let $Z = Z_R \cup Z_L \subset X$. Set

$$\mathcal{I} = \{A \subset \Gamma : \mathrm{cls}\, A \cap Z = \emptyset\} = \mathcal{I}_R \cap \mathcal{I}_L.$$

Set $\mathcal{F} = \{B \subset \Gamma : B^c \in \mathcal{I}\} = \{B \subset \Gamma : \mathrm{cls}\, B \supset Z\}$. Clearly then $\mathcal{I}$ is an ideal and $\mathcal{F} = \mathcal{F}_R \cap \mathcal{F}_L$ is a filter.

4. A subset $A \subset \Gamma$ is called *right translation-finite* (RTF for short) if for every infinite $D \subset \Gamma$ there is a finite $F \subset D$ such that $\cap_{\delta \in F} A\delta^{-1}$ is finite. We denote by $\mathcal{I}_{RTF}$ be the collection of RTF subsets of Γ. We say that a subset $B \subset \Gamma$ is *co-right-translation-finite* (CRTF) if $B^c = \Gamma \setminus B$ is RTF and denote the collection of CRTF

sets by $\mathfrak{F}_{RTF}$. Thus a subset $B \subset \Gamma$ is CRTF if for every infinite subset $D \subset \Gamma$ there is a finite subset $F \subset D$ such that $\cup_{\delta \in F} B\delta^{-1}$ is co-finite in Γ. These notions have obvious left analogues, LTF subsets of Γ, $\mathfrak{I}_{LTF}$, etc. Following Ruppert we say that elements of $\mathfrak{I}_{TF} := \mathfrak{I}_{LTF} \cap \mathfrak{I}_{RTF}$ are *translation-finite sets* (TF).

5. We let $\mathfrak{I}_W$ be the collection of sets $A \subset \Gamma$ such that $\operatorname{cls} A$ is an open subset of Γ^{WAP} with $\operatorname{cls} A \cap \operatorname{cls} Y^2 = \emptyset$. Then $\mathfrak{F}_W = \{A^c : A \in \mathfrak{I}_W\}$.
6. We say that $A \subset \Gamma$ is a *W-interpolation set* if $\operatorname{cls} A \subset \Gamma^{WAP}$ is an open subset of Γ^{WAP} which is homeomorphic to βA. We let $\mathfrak{I}_{IW}$ denote the collection of W-interpolation sets, and let $\mathfrak{F}_{IW} = \{A^c : A \in \mathfrak{I}_{IW}\}$.

Recall the following theorems of Ruppert (Theorem 7 and Proposition 13 in [3]).

2.2. **Theorem.** 1. *$\mathfrak{I}_{TF}$ is an ideal and $\mathfrak{I}_{TF} = \mathfrak{I}_W = \mathfrak{I}_{IW}$.*
2. *Every infinite subset of Γ contains an infinite TF subset.*

Ruppert's main tools in analyzing the TF property were the universal WAP compactification of Γ and Grothendieck's double limit characterization of WAP functions. Our approach is through the Čech-Stone compactification of Γ and the combinatorial definition of the product of ultrafilters.

2.3. **Theorem.** 1. *$\mathfrak{F}_R = \mathfrak{F}_{RTF}$, in particular $\mathfrak{F}_{RTF}$ is a filter.*
2. *$\mathfrak{F}_L = \mathfrak{F}_{LTF}$, in particular $\mathfrak{F}_{LTF}$ is a filter.*
3. *$\mathfrak{F} = \mathfrak{F}_{TF} = \mathfrak{F}_{RTF} \cap \mathfrak{F}_{LTF}$, hence*

$$\mathfrak{I}_{TF} = \mathfrak{I}_{LTF} \cap \mathfrak{I}_{RTF} = \mathfrak{I}_L \cap \mathfrak{I}_R = \mathfrak{I} = \mathfrak{I}_W = \mathfrak{I}_{IW}$$

Proof. We prove the two inclusions of claim (1) below. The claim (2) then holds by symmetry and claim (3) is obtained by taking the appropriate intersections and applying Ruppert's theorem.

Side 1: We first show that $\mathfrak{F}_{RTF} \subset \mathfrak{F}_R$. Consider $B \in \mathfrak{F}_{RTF}$ and suppose $A \subset \Gamma$ has the property that there are $p, q \in X$ with $A \in pq$; i.e. $Ap^{\leftarrow} := \{\gamma \in \Gamma : A\gamma^{-1} \in p\} \in q$. Then $|Ap^{\leftarrow}| = \infty$ and by assumption there is a finite subset $F \subset Ap^{\leftarrow}$ such that $\cup_{\delta \in F} B\delta^{-1}$ is cofinite in Γ. As p is an ultrafilter this implies that for some $\delta \in F$ we have $B\delta^{-1} \in p$. Now, as both $B\delta^{-1}$ and $A\delta^{-1}$ are in p so is $(A \cap B)\delta^{-1}$. In particular we conclude that

$A \cap B \neq \emptyset$. This discussion shows that for any two ultrafilters p, q in X their product pq is in $\mathrm{cls}\, B$; hence $\mathrm{cls}\, B \supset Z_R$, i.e. $B \in \mathcal{F}_R$.

Side 2: Next we show that $\mathcal{F}_R \subset \mathcal{F}_{RTF}$. Suppose then that $A \subset \Gamma$ is not in $\mathcal{F}_{RTF}$; i.e. there is an infinite $D \subset \Gamma$ such that for every finite $F \subset D$ we have $|(AF^{-1})^c| = |\Gamma \setminus \cup_{\delta \in F} A\delta^{-1}| = \infty$. Clearly then the collection of sets of the form $(AF^{-1})^c$, with $F \subset D$ finite, is a filter, say $\mathcal{L}$, on Γ. Choose some ultrafilter $p \supset \mathcal{L}$. Now choose an ultrafilter q with $D \in q$. We will show that $A \notin pq$, whence $A \notin \mathcal{F}_R$, as required.

Assuming $A \in pq$ we have $Ap^{\leftarrow} = \{\gamma \in \Gamma : A\gamma^{-1} \in p\} \in q$. However if $\delta \in D$ then $(A\delta^{-1})^c \in \mathcal{L}$, hence $(A\delta^{-1})^c \in p$, hence $A\delta^{-1} \notin p$, hence $D^c \supset Ap^{\leftarrow} \in q$, hence $D^c \in q$. This is a contradiction and we conclude that indeed $A \notin pq$. □

2.4. **Theorem.** 1. *We have* $\pi^{-1}(\mathrm{cls}\, Y^2) = Z = Z_R \cup Z_L$, *hence* $\pi^{-1}(Y \setminus \mathrm{cls}\, Y^2) = X \setminus Z$.

2. *The restriction of* π *to the open dense subset* $X \setminus Z$ *of* X *is a homeomorphism from* $X \setminus Z$ *onto* $Y \setminus \mathrm{cls}\, Y^2$.

Proof. **Step 1:** Given $y \in U \subset (\Gamma^{WAP} \setminus \mathrm{cls}\, Y^2)$, where $y \in Y$ and U is an open subset of Γ^{WAP}, let V be an open subset of Γ^{WAP} such that $y \in V \subset \mathrm{cls}\, V \subset U$. The set $\tilde{V} = \pi^{-1}(V)$ is an open subset of $\beta\Gamma$ such that $\mathrm{cls}\, \tilde{V} \cap Z = \emptyset$ (since π is a homomorphism of semigroups we have $\pi(X^2) = \pi(X^{\cdot 2}) = Y^2$, for both right and left semigroup structures on $\beta\Gamma$). Let $A = \Gamma \cap \tilde{V}$, then $\mathrm{cls}_{\beta\Gamma} A = \mathrm{cls}\, \tilde{V}$ and therefore $A \in \mathcal{I}$. By Theorem 2.3.3 we have $A \in \mathcal{I}_{TF}$ and then, by Theorem 2.2, $A \in \mathcal{I}_W$. We conclude that $\mathrm{cls}_{\Gamma^{WAP}} A$ is a clopen neighborhood of y which is contained in U. Thus we have shown that the collection of sets of the form $\mathrm{cls}\, A$ with $A \in \mathcal{I}_{TF}$, is a basis for the topology on $\Gamma^{WAP} \setminus \mathrm{cls}\, Y^2$.

Step 2: If A is any set in $\mathcal{I}_{TF}$ then again by Theorem 2.2, $A \in \mathcal{I}_W = \mathcal{I}_{IW}$ and we conclude that $\mathrm{cls}\, A$ is a clopen subset of Γ^{WAP} which is homeomorphic to βA. By the universality of βA it follows that $\pi : \mathrm{cls}_{\beta\Gamma} A \to \mathrm{cls}_{\Gamma^{WAP}} A$ is a homeomorphism.

Step 3: Again if A is any set in $\mathcal{I}_{TF}$ then, by Theorem 2.2, $A \in \mathcal{I}_W$ and we conclude that $\mathrm{cls}\, A$ is a clopen subset of Γ^{WAP}. We claim that $\pi^{-1}(\mathrm{cls}_{\Gamma^{WAP}} A) = \mathrm{cls}_{\beta\Gamma} A$. Clearly $\mathrm{cls}_{\beta\Gamma} A \subset \pi^{-1}(\mathrm{cls}_{\Gamma^{WAP}} A)$. Conversely, if $p \in \beta\Gamma$ with $\pi(p) = y \in \mathrm{cls}_{\Gamma^{WAP}} A$, let $p = \lim \gamma_\nu$ for a net $\gamma_\nu \in \Gamma$. Then $y = \pi(p) = \lim \pi(\gamma_\nu) = \lim \gamma_n u$ and, as by assumption the set $\mathrm{cls}_{\Gamma^{WAP}} A$ is a clopen subset of Γ^{WAP}, it follows that eventually $\gamma_\nu \in A$. Thus we have $p \in \mathrm{cls}_{\beta\Gamma} A$ as claimed.

Step 4: By Proposition 13 of [3] (Theorem 2.2.2), every infinite subset $B \subset \Gamma$ contains an infinite subset $A \subset B$ with $A \in \mathcal{I}_{TF}$. In view of step 1 above this shows that the set $Y \setminus \operatorname{cls} Y^2$ is a dense open subset of Y.

Step 5: Summing up we have shown that (i) the collection of clopen sets $\{\operatorname{cls}_{\Gamma^{WAP}} A : A \in \mathcal{I}_{TF}\}$ forms a basis for the topology on $\Gamma^{WAP} \setminus \operatorname{cls} Y^2$, (ii) for each $A \in \mathcal{I}_{TF}$, $\pi^{-1}(\operatorname{cls}_{\Gamma^{WAP}} A) = \operatorname{cls}_{\beta\Gamma} A$ and moreover (iii) $\pi : \operatorname{cls}_{\beta\Gamma} A \to \operatorname{cls}_{\Gamma^{WAP}} A$ is a homeomorphism. These facts together with the fact that $Y \setminus \operatorname{cls} Y^2$ is a dense open subset of Y prove the assertions of Theorem 2.4. □

3. Divisible properties, IP and D sets

In [1] a collection $\mathcal{P}$ of subsets of Γ is called a *divisible property* if

(i) $\emptyset \notin \mathcal{P}$ and $\Gamma \in \mathcal{P}$,

(ii) $\mathcal{P}$ is hereditary upward (i.e. $A \in \mathcal{P}$ and $B \supset A$ imply $B \in \mathcal{P}$ and

(iii) if $A \in \mathcal{P}$ is a union $A = A_1 \cup A_2$ then at least one of the sets A_1 and A_2 is in $\mathcal{P}$.

A collection $\mathcal{P}$ is *divisible* iff the collection $\mathcal{I} = \{A \subset \Gamma : A \notin \mathcal{P}\}$ is an ideal iff the *dual* collection $\mathcal{F} = \mathcal{P}^* = \{A \subset \Gamma : A \cap B \neq \emptyset, \ \forall B \in \mathcal{P}\}$ is a filter. When $\mathcal{F}$ is a filter of subsets of Γ the compact (nonempty) subset $K = \bigcap \{\operatorname{cls} A : A \in \mathcal{F}\} \subset \beta\Gamma$ is called the *kernel* of $\mathcal{F}$. Conversely, any compact subset $K \subset \beta\Gamma$ defines a filter

$$\mathcal{F} = \{A \subset \Gamma : \operatorname{cls} A \supset K\}.$$

The correspondence $\mathcal{F} \leftrightarrow K$ is one to one and we note that

$$\mathcal{I} = \{A \subset \Gamma : \operatorname{cls} A \cap K = \emptyset\} \quad \text{and} \quad \mathcal{P} = \{A \subset \Gamma : \operatorname{cls} A \cap K \neq \emptyset\},$$

are the corresponding ideal and divisible properties respectively.

Expressed explicitly the divisible property which corresponds to the ideal of RTF-sets is the following one: a subset $A \subset \Gamma$ is not right translation finite, *an NRTF-set*, if there exists an infinite subset $D \subset \Gamma$ such that for every finite subset $F \subset D$ the corresponding intersection $\bigcap_{\delta \in F} A\delta^{-1}$ is infinite. NLTF-sets are defined similarly and a set A is NTF if if there exists an infinite subset $D \subset \Gamma$ such that for every finite subset $F \subset D$ at least one of the two corresponding intersections $\bigcap_{\delta \in F} A\delta^{-1}$ and $\bigcap_{\delta \in F} \delta^{-1} A$ is infinite. In this terminology Theorem 2.3 is stated as follows:

3.1. **Theorem.** *The properties NRTF, NLTF and NTF are divisible with corresponding kernels* Z_R, Z_L *and* Z *respectively.*

Note however that the ideal $\mathcal{I}_W$ is not what we call in [1] the collection of interpolation sets of the algebra $WAP(\Gamma)$, as in Definition 2.1.6 we postulate that $A \in \mathcal{I}_W$ when it is a $WAP(\Gamma)$ interpolation set which additionally satisfies the requirement that $\mathbf{1}_D \in WAP(\Gamma)$. In [1] (Corollary 5.3.2) we have shown that the collection $\mathcal{J}$ of WAP-interpolation sets has the property that if $\Gamma = \bigcup_{i=1}^n A_i$ then at least one of the sets A_i is not in $\mathcal{J}$. Let Γ_{dis}^{WAP} denote the *universal totally disconnected semitopological compactification* of Γ. It is obtained as the quotient $\Gamma^{WAP}/\sim$ of Γ^{WAP} by the equivalence relation: $x \sim y \iff x$ and y lie in the same connected component. Let $WAP_{dis}(\Gamma)$ denote the corresponding C^*-algebra.

3.2. **Problem.** (a) Is the collection of $WAP(\Gamma)$-interpolation sets an ideal ?

(b) Is the collection of $WAP_{dis}(\Gamma)$-interpolation sets an ideal ?

For simplicity let us assume next that Γ is abelian. We will denote the group operation by $+$ but keep the notation $(p,q) \mapsto pq$ for the semigroup operation on $\beta\Gamma$. Recall that a subset A of Γ is a *D-set* if there is an infinite sequence $\{\gamma_i\}_{i=1}^\infty \subset \Gamma$ such that for every $i \neq j$ at least one of the elements $\gamma_i - \gamma_j$ or $\gamma_j - \gamma_i$ is in A. The subset A is called an *IP-set* if there is an infinite sequence $\{\gamma_i\}_{i=1}^\infty \subset \Gamma$ such that for every finite sequence $i_1 < i_2 < \cdots < i_n$ the element $\gamma_{i_1} + \gamma_{i_2} + \cdots + \gamma_{i_n}$ is in A. It is well known that Hindman's theorem is equivalent to the fact that the collection of IP-sets is a divisible property with the set $K = \operatorname{cls}\{v \in X : v^2 = v\}$ (the closure of the set of idempotents in X) as its kernel. Obviously $K \subset Z$. It is easy to see that every IP-set is also a D-set.

The filter which corresponds to the IP-sets is the collection of IP*-sets:

$$\{A \subset X : \operatorname{cls} A \supset K\} = \{A \subset \Gamma : A \cap B \neq \emptyset,\ \forall \text{ IP-set } B\}.$$

Similarly the filter which corresponds to the D-sets is the collection of D*-sets:

$$\{A \subset X : \operatorname{cls} A \supset K\} = \{A \subset \Gamma : A \cap B \neq \emptyset,\ \forall \text{ D-set } B\}.$$

The fact that the collection of D-sets is a divisible property is equivalent to Ramsey's theorem and in [1] we have identified the kernel of this divisible property as the following closed subset $L \subset X$. Define the set $V \subset X$ as follows: $p \in X$ is in V iff there is an element $q \in X$ and a net γ_α in Γ such that $\lim \gamma_\alpha = q$ and $p = \lim \gamma_\alpha^{-1} q$. Now put $L = \operatorname{cls} V$.

It is easy to see that $V \subset X^2$, whence $L \subset Z$. Thus the identifications of the kernels K and L, together with Theorem 2.3, immediately lead to the following corollary.

3.3. **Corollary.** *Every CTF-set (i.e. the complement of a TF-set) is a D^*-set and a fortiori an IP^*-set.*

References

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