TEL AVIV UNIVERSITY
Department of Computer Science
0368.4281 – Advanced topics in data structures
Spring Semester, 2001/2002

Homework 4, June 5, 2001

Due on Wed June 19. Don't forget to keep a copy of the homework.

1. Extend the data structure described in class for the "order maintenance" problem to a more efficient data structure in which every operation costs $O(1)$ amortized time.

2. We build a tree dynamically using the operation $insert(x, y)$ that adds node x as a child of node y which is already in the tree. At any time between the insert operations we would like to be able to answer queries of the form $ancestor(x, y)$: Is x an ancestor of y in the tree ? Describe an efficient algorithm for the problem.

3. Extend the fully persistent queues presented in class such that they support the following operations.
1) $Q' = inject(Q, x, d)$ that produce Q' by inserting element x immediately before the d last elements in the queue Q.
2) $(x, Q') = pop(Q, d)$ that produce a queue Q' from Q by taking out the dth element, x, from the begining of Q.
What are the time and space complexities of your data structure ?

4. Describe an implementation of fully persistent heaps that support the operations *find-min*, *insert*, and *delete-min*. Make your implementation as efficient as you can. What are the time and space complexities of your data structure ?

5. We perform a sequence of m operations on an heterogenous 2-4 finger search tree (i.e. a finger search tree without level links). Out of these m operations k are concatenations where the ith concatenation concatenates two trees such that the smaller among them contains n_i elements. The other $m - k$ operations are find, insert, delete, and split. The j-th operation among these operations is performed on an element at distance d_j from the beginning of a list containing n_j elements. Prove that using the implementations of the operations as described in class one can perform the sequence in

$$O(\sum_{i=1}^{k} \log(n_i) + \sum_{j=1}^{m-k} \log(\min\{d_j, n_j - d_j\}))$$

time.

6. Improve the bound in the previous question to

$$O(k + \sum_{j=1}^{m-k} \log(\min\{d_j, n_j - d_j\}))$$

time.