TEL AVIV UNIVERSITY
Department of Computer Science
0368.4281 – Advanced topics in data structures
Spring Semester, 2003/2004

Homework 4, May 24, 2003

Due on Mon. June 7

1. We say that a flow f is acyclic if there is no cycle with positive flow along it. Given a flow f from s to t in a directed graph $G = (V, E)$ show how to get from f an acyclic flow f' such that $|f| = |f'|$ (try to get an algorithm that runs in $O(m \log n)$ time.).

2. We define an *expression* to be a legal arithmetic fully parenthesized expression containing the binary operators $+$ and $*$ and integers. For example $(((2+4)*3)+(5*3))$ is an expression. We can describe an expression as a binary tree with the numbers at the leaves and each internal node corresponds to an operator. Figure 1 shows the binary tree which corresponds to the expression $(((2 + 4) * 3) + (5 * 3))$. Note that the binary tree representation of an expression is unique.

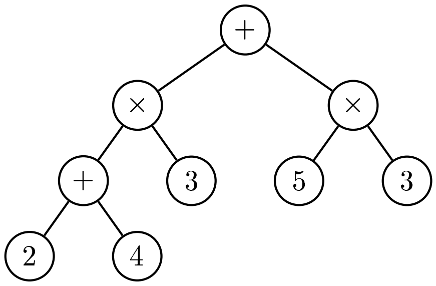

Figure 1: The tree corresponding to the expression $(((2 + 4) * 3) + (5 * 3))$.

Describe a data structure representing expression trees that supports the following operations.

1. $MakeLeaf(x)$: Creates a new tree containing a single leaf with the value x.
2. $Evaluate(T)$: Returns the value of the expression represented by T.
3. $Change(v, T, x)$: v is a leaf in the expression tree T. The operation changes the value associated with v to x.
4. $Combine(T_1, T_2, op)$: T_1 is an expression tree representing the expression e_1, T_2 is an expression tree representing e_2, and op is either $+$ or $*$. Returns a new tree T representing the expression e_1 *op* e_2.

Please analyze your data structure. Try to get a solution where all operations take $O(\log n)$ time.

3. Show how to extend dynamic trees to support an operation called $evert(v)$. This operation should change the root of the tree containing v to be v. In other words we change the direction of the edges on the path from v to the previous root of the tree.

4. The *lower envelope* of the set of lines S is the piecewise linear function $f(x) = \min\{ax + b \mid (y = ax + b) \in S\}$. A vertex of the lower envelope is a point $(x, f(x))$ which is an intersection of two lines in S.

We start with an empty set of lines S and perform a sequence $\sigma_1, \ldots, \sigma_n$, where each σ_i is either an insertion of a line to S or a deletion of a line from S. Let E_i be the set of vertices on the lower envelope of the lines in S after we performed $\sigma_1, \ldots, \sigma_i$. Let $E = \cup_{i=1}^{n} E_i$. Prove an upper bound as tight as you can on $|E|$.

5. Extend the data structure described in class for the "order maintenance" problem to a more efficient data structure in which every operation costs $O(1)$ amortized time.