The Congruence Subgroup Problem for Free Metabelian Groups

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Abstract

In its classical setting, the congruence subgroup problem (CSP) asks: Does every finite index subgroup of $G = GL_n(\mathbb{Z})$ contain a principal congruence subgroup, i.e. a subgroup of the form $G(m) = \ker(GL_n(\mathbb{Z}) \to GL_n(\mathbb{Z}/m\mathbb{Z}))$ for some $m \in \mathbb{N}$? This is equivalent to the question: Is the congruence map $\widehat{GL_n(\mathbb{Z})} \to GL_n(\hat{\mathbb{Z}})$ injective? So the modern version of the problem is: What is $C_n = \ker(\widehat{GL_n(\mathbb{Z})} \to GL_n(\hat{\mathbb{Z}}))$? It is known that there is a dichotomy between the case $n = 2$, in which the kernel $C_2 = \hat{F}_\omega$ is huge and equal to the free profinie group on countable number of generators, and the cases $n > 2$, in which the kernel is trivial.

Viewing $GL_n(\mathbb{Z}) = Aut(\mathbb{Z}^n)$ as the automorphism group of the free abelian group on n generators, one can generalize the CSP as follows: Let Δ be a finitely generated group. What is $C(\Delta) = \ker(\widehat{Aut(\Delta)} \to Aut(\hat{\Delta}))$? Considering this generalization, very few results are known when Δ is non-abelian. For example, only in 2001 Asada proved, using tools of algebraic geometry, that $C(F_2)$ is trivial, and the CSP for $F_{n>2}$ is still unsettled.

On the talk, we will discuss some non-abelian cases, and in particular we will focus on what happens in the case in which Δ is a finitely generated free metabelian group. We will show that in this case we have a dichotomy between $n = 2, 3$ and $n > 3$.