A tale of two roads[1]

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Length bias is a well known phenomenon among statisticians. It refers to the fact that individuals that stay in a system for a longer time have higher chances to be sampled. Some examples are: The passengers of an airline will not understand why the company is loosing money; their experience shows that the average occupancy of a plane of this airline is high (most passengers however, fly in flights that are more crowded than the average!) Similarly, students may not understand why their tuition does not cover the university's expenses; their classes were almost always very large. Bus passengers may complain that busses are too crowded, though the bus company claims that on the average the bus is used for less than half of its capacity.

Now, let us describe a related puzzle. Two roads connect cities A and B. The roads are of identical length and quality. Every morning, about 2500 cars commute from A to B. A rider chooses the road to be used, and once on it cannot change this decision. The travel time increases with the load on the road, and therefore a rider wants to be on the less crowded road. However, there is nothing to do about the following fact: **A majority of riders use the more crowded road!** This is always true, no matter how decisions are made. Now, each driver independently decides whether to use A or B, and in our case, they have no information about the number of others using the roads that day (in other cases they may use past experience and this may lead to an equilibrium with about the same number of drivers in each road). Let Mr. Name be one of these drivers. We meet him just prior to his departure, after he had made his decision. We can provide him with the following reasoning; listen Mr. Name, our mathematical calculations indicate that the road you have chosen (no matter which one) is going to be more crowded than the other road with probability greater than 0.5. (Recall that a majority of riders use the more crowded road.) Moreover, the number of cars in the more crowded road is greater than that of the other road by about 40 on the average.[4] This may represent a considerable difference in driving time. Our advice to you is that you reverse your decision, improving your chances to be on the less crowded, faster, road. TRUE?

SOLUTION: Of course not. It is true that the expected difference between the more crowded road and the other one is about 40. It is also true that the chances are greater than 0.5 that Mr. Name chooses the bad alternative. But there is nothing to do about it. The expected number of passengers in the road he chooses is grater by exactly one than that of the alternative. If he changes his decision, the *conditional* expected number of users will change too, decreasing by one in the road he had chosen originally, increasing by one in the other one.

[1] A version of this puzzle, edited by Barry Nalebuff, appeared in "Puzzles: Queues, Coups and More" *Journal of Economic Perspectives* (Spring 1990).

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[4] Let X_A and X_B be the number of passengers in roads A and B, respectively. Each variable is binomial with $p = 0.5$ and $n = 2500$. Using the Normal approximation with variance 625, $E(|X_A - X_B|) = 2\sqrt{625}\sqrt{2/\pi} \approx 40$. [We use the fact that the expected absolute value of a standard normal variable is $\sqrt{2/\pi}$.]