Test for Optimality.

According to the theory of the simplex method with bounded variables (see Chapter 18), if the non-basic variables satisfy the following conditions:

(a) they are all at either their upper or lower bounds,

(b) their corresponding $\bar{c}_{ij}$ and $\bar{c}'_{hj} \geq 0$, if they are at their lower bound, and

(c) their corresponding $\bar{c}_{ij}$ and $\bar{c}'_{hj} \leq 0$ if they are at their upper bound,

then the solution is optimal and the algorithm terminates. Otherwise there are $\bar{c}_{ij}$ or $\bar{c}'_{hj}$ for which (b) or (c) does not hold. In which case an increase or decrease (depending on whether the sign is negative or positive) in the corresponding variable is allowed; we will call these (i, j) or (h, j) combinations *out-of-kilter*; let the largest $\bar{c}_{ij}$ or $\bar{c}'_{hj}$ among them in absolute value be denoted by $\bar{c}_{rs}$ or $\bar{c}'_{rs}$.

Step 5. Leaving all non-basic entries fixed except for the value of the variable corresponding to the (r, s) determined in Step 4, modify the value of x_{rs} (or y_{rs}), if not at its upper bound, to

$$x_{rs} + \theta \text{ (or } y_{rs} + \theta) \text{ if } \bar{c}_{rs} < 0 \text{ (or } \bar{c}'_{rs} < 0) \tag{21}$$

or, if not at its lower bound, to

$$x_{rs} - \theta \text{ (or } y_{rs} - \theta) \text{ if } \bar{c}_{rs} > 0 \text{ (or } \bar{c}'_{rs} > 0) \tag{22}$$

where $\theta \geq 0$ is unknown for the moment, and recompute the values of the basic variables as linear functions of θ. Choose the value of $\theta = \theta^*$ at the largest value possible consistent with keeping all basic variables (whose values now depend on θ) between their upper and lower bounds; in the next cycle correct the values of the basic variables on the assumption $\theta = \theta^*$.

Also, if at the value $\theta = \theta^*$ one (or more) of the basic variables attains its upper or lower bound, in the next cycle drop any one of these variables (never drop more than one) from the basic set and box the variable x_{rs} instead. Should it happen that x_{rs} or y_{rs} attains its upper or lower bound at $\theta = \theta^*$, the set of basic variables is the same as before; their values, however, are changed to allow x_{rs} or y_{rs} to be fixed at its new bound.

Start the next cycle of the iterative procedure by returning to Step 3.

28-2. NUMERICAL SOLUTION OF THE ROUTING PROBLEM

For our starting solution in Table 28-2-IV, cycle o, we used for values of x_{ij} the best solution assuming fixed demands equal to the expected values of the distribution[3] shown in Table 28-2-II. These x_{ij} will meet the expected

[3] In the humorous parody by Paul Gunther [1955-1] entitled "Use of Linear Programming in Capital Budgeting," *Journal of the Operations Research Society of America*, May, 1955, it will be recalled that Mrs. Efficiency wondered why Mr. O. R. did not start out with a good guess. In this chapter you will note that we followed Mrs. Efficiency's suggestion and have started by guessing at the final solution rather than going through the customary use of artificial variables and a Phase I of the simplex process.

LINEAR PROGRAMMING AND EXTENSIONS

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