What risk should a selfish partner take in order to save the life of a non-relative, selfish friend?

A stochastic game approach to the prisoner's dilemma

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Abstract

A model of cooperation versus defection in a sequence of games is analyzed under the assumptions that the rules of the games are randomly changed from one encounter to another, that the decisions are to be made at each situation anew, according to the specific rules of the specific local game, and that the outcome of each such game affects the ability of a player to participate and, thus cooperate (if in its own interest) in the next game. Players wish to maximize their total payoffs at the entire supergame. Under plausible assumptions it is shown that all Evolutionarily Stable Strategy (ESS) of the supergame determine cooperation over a non-degenerate range of situation-determining encounters of the P.D. type - A moderately altruistic cooperation is selected because it enables the survival of a potential partner for future cooperation. The model also explains the evolution of gratitude, rather than the assumption thereof, and predicts a qualitative difference between partnership altruism and kin altruism.

Introduction

It seems that an appropriate question to begin the present article with may be "Why should selfish partner take any risk in order to save the life of a non-relative selfish friend?" (Instead of what risk he should take, as mentioned in the title of this paper). An established fact, though,
that non-relatives who have a history of cooperation (in this context "friends") do tend to take limited risks in order to help each other. It is precisely this fact that raises the question that we now refer to as the paradox of the prisoner’s dilemma.

In the last three decades, many answers were given to this apparent paradox. Most, if not virtually all of them, were based on the simple idea (e.g., Rapoport, 1967) that human beings (and, most likely, other social animals) are bound to play again (at least with high probability) with the same player they are now encountering. It is beyond the scope of this article to summarize the bulk of literature, devoted to the analysis of the repeated game of the prisoner’s dilemma type, or to list the difficulties which still face this model. (For a general survey and references, see Axelrod, 1980).

It seems to us, however, that most of these difficulties stem from one unrealistic assumption, common both to the bulk of theoretical models and laboratory experiments, namely that individuals are assumed to play the same game repeatedly under exactly the same conditions.

It seems, on the other hand, that all natural examples of cooperation involve a rather multidimensional situation in which two (or more) individuals meet each other repeatedly, being faced with a somewhat different situation each time. In such natural cases, a natural question may not be whether or not to cooperate in the next encounter. More likely, an individual has to decide under what condition or in what situation to cooperate.

Repeated encounters under a varied situation

Let us start by imagining an encounter of two shepherds with a lion. Sticking together, the two shepherds have the best chance to survive, yet each of them may drastically increase his chance by escaping and letting the other die. An observer who is familiar with the repeated game model is likely to be aware of the fact that encounters with lions occurred repeatedly in the past and are likely to repeat in the future and to explain their behavior on this basis. Yet such encounters may be too rare and the immediate advantage of escaping may be too big as to account for the cooperative behavior of the partner.

The dilemma may partly be resolved if the observer is aware of a much more frequent situation in which the same two shepherds must cooperate in rescuing their herd from frequent attacks of, say, jackals. Indeed, cooperating against an invading jackal involves no serious risk and thus represents no theoretical difficulty. Hence, it is likely to escape the eye of the kin-minded, theoretically oriented observer. But as cooperation in defining the herd against jackals becomes a key factor in the shepherds’ lives, the death of one of them presents the other with a serious problem in the very near future. This is not because the deserted, now dead, partner will “remember” and desert the defector in the future, say a year from now, when perhaps the next encounter with a lion takes place, but because he will not be there to help even if it is in his best interest to, and he will certainly not be there to help the following day in the daily fight against jackals. Hence, it may be in his partner’s best interest to help him now, taking a big risk against a lion, or it may not, depending on the parameters. But it becomes more likely if, in addition to the risk of attacks by jackals and lions, we also take into consideration other risks, say attacks by wolves.

The incentive of saving your partner’s life in order that he will be there to help you against jackals the next day, may not be enough to take as big a risk as fighting a lion. But it is sufficient to make it worthwhile to take the smaller risk of cooperating against a wolf. However, if encounters with wolves are sufficiently common (perhaps not as common as encounters with jackals), the life and welfare of one partner becomes crucially important to the other, since a wolf, unlike a jackal, may be quite dangerous when faced alone. It may be sufficiently important as to risk one’s life cooperating in a struggle against a lion, or perhaps in the case of a not-too-frightening lion, but then we come back to the more realistic question of under what circumstances to cooperate rather than whether to cooperate.

We believe that the evolution of cooperation must have started in the simplest and most natural way in a limited (and, one has to admit, theoretically uninteresting) range of situations wherein cooperation is of a direct advantage to both participants. Thus, the appropriate supergame-strategy for this initial stage is "cooperate at any future encounter in which you cannot gain by exclusive defection". However, once such a strategy is established in a population, the very survival (and, maybe, well-doing) of your present time partner until the next possible encounter becomes important to your long-term survival (see, for comparison, Eshel & Cohen, 1976, Eshel & Motro, 1981; Motro, 1988). Indeed, his death throughout the present encounter will surely
prevent him from any future cooperation even under those situations wherein he will have all the incentives to cooperate.

It can, therefore, be shown that an individual can increase his long-term success by a slight increase of the range of situations under which he is willing to cooperate. More specifically, it is in his best interest to cooperate in such situations in which, by paying a sufficiently small price, he can substantially increase the success of his partner, so as to enable him to cooperate in the future (at least under situations wherein cooperation will be advantageous to both parties). Yet as anyone in the population increases his own range of cooperation, the higher the probability becomes that a present-encounter-partner, if he survives, will cooperate in the future and, thus, the higher the incentive to overcome the temptation of short term gain through defection.

In the present work we suggest a model for a natural process in which individual selection operates to increase the range of cooperation in a continuous way from selfish cooperation to an evolutionary stable range of cooperation which includes some or all possible encounters of the prisoner’s dilemma type. For this, neither kinship nor ability to reciprocate are required (though we do believe that these are likely to be enhancing factors in real situations).

We start by describing in the next section, a closely related, previously published model developed under a somewhat restrictive assumption of independence between death events of the two players. For detailed mathematical proofs, the reader is referred to Eshel and Weinshall (1988). We then summarize some new results, stemming from the relaxation of this assumption. Detailed mathematical analysis of this case will be published elsewhere.

### Repeated game with a random encounter rule - The case of independent death events

Assume that two players encounter each other, repeatedly, a random number of times so that at the end of each encounter there is a probability \( p > 0 \) for having another one. Assume a general matrix of a symmetric two-person encounter in which each of the players can either cooperate or defect:

<table>
<thead>
<tr>
<th></th>
<th>cooperate</th>
<th>defect</th>
</tr>
</thead>
<tbody>
<tr>
<td>cooperate</td>
<td>( X_1, X_2 )</td>
<td>( X_3, X_4 )</td>
</tr>
<tr>
<td>defect</td>
<td>( X_5, X_6 )</td>
<td>( X_7, X_8 )</td>
</tr>
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where the parameters \( X = (X_1, ..., X_8) \) are random variables, drawn from a four dimensional distribution, \( F \), independently of the past. By choosing the term "cooperation" for the first strategy, however, we mean that

\[
X_1 > X_3 \text{ and } X_2 > X_4
\]

(i.e., by cooperating, a player always helps his partner),

\[
X_2 > X_3
\]

(i.e., mutual cooperation is always in the Pareto's optimum of the encounter), and

\[
X_3 > X_2
\]

(i.e., if only one player defects, then his reward will be higher than that of his cooperating partner).

We assume a positive probability for encounters of the prisoner’s dilemma type, i.e.,

\[
p(D) > 0 \text{ where } D = \{X \mid X_1 > X_3, X_2 > X_4\}
\]

We also assume, however, a positive probability for encounters in which cooperation is of immediate self reward, i.e.,

\[
p(R) > 0 \text{ where } R = \{X \mid X_1 > X_2, X_3 > X_4\}
\]

We assume, moreover, that \( F \) has positive density \( f \) over a convex set of parameters, \( \Omega \), including at least part of the boundary \( \{X \mid X_1 = X_3 > X_2 = X_4\} \) between \( R \) and \( D \).

At each stage of the supergame each player possesses full knowledge of the present situation (i.e., about the realization of \( X \)). However, we assume no memory so that a pure strategy is a measurable set \( G \) of situations (game-matrices) over which the player is willing to cooperate. A mixed strategy is a measurable function \( \Gamma : \Omega \rightarrow [0,1] \), determining the probability \( \Gamma(a) \) that a player will cooperate in situation \( a \in \Omega \), where \( \Omega \) is the space of all payoff matrices.

If player 1 (i = 1,2) chooses the strategy \( \Gamma \), then the survival probability of player 1 during
that at any single encounter, player 1 survives and player 2 dies. Employing the stationary property of the sequence of games, we know that the survival probability of player 1 until the end of the sequence, conditioned on this opponent’s death at one stage of the supergame is

$$\sum_{k=0}^{n} p^k q^{n-k} = \frac{q}{1 - p \lambda} \quad (3.10)$$

The unconditioned survival probability of player 1 to the end of the sequence is, therefore,

$$\nu(\Gamma_1, \Gamma_2) = \frac{q}{1 - p\lambda} + \left[ \frac{1}{1 - p\lambda} \cdot \frac{s_1 (1 - s_2)}{s_1 s_2} \cdot \frac{q}{1 - p\lambda} \right]$$

where $s_1$, $s_2$ and $\lambda$ are given by (3.6)-(3.8). By simple algebraic manipulation one readily gets

$$\nu(\Gamma_1, \Gamma_2) = \frac{q}{1 - p\lambda} (1 + p\psi) \quad (3.11)$$

where

$$\psi = \psi(\Gamma_1, \Gamma_2) = \psi(s_1, s_2) = \frac{s_1 - \lambda}{1 - s_1 s_2} \quad (3.12)$$

Hence, the game is determined by the attempt of player 1 to choose a strategy $\Gamma_1$ that will maximize $\nu(\Gamma_1, \Gamma_2)$ against $\Gamma_2$ and of player 2 to choose a strategy $\Gamma_2$ that will maximize $\psi(\Gamma_2, \Gamma_1)$ against $\Gamma_1$.

From (3.12) it immediately follows that indeed

$$\frac{\partial \psi(s_1, s_2)}{\partial s_2} > 0 \quad (3.13)$$

i.e., at each encounter each player will gain by increasing his own survival probability, but as long as $s_1 > \lambda$ (which is true for any reasonable strategy-choice since player 1 can guarantee the survival probability $s_1 = \lambda$ by the apparently non-optimal strategy of never cooperating) also
\[
\frac{\partial \psi(s_1, s_2)}{\partial s_2} > 0
\]

(3.14)

which means that it is in the interest of each player to increase the survival probability of his partner. Note, however, that since both \( s_1 \) and \( s_2 \) are functions of \( \Gamma_1 \) and \( \Gamma_2 \) and since player 1 can only change \( \Gamma_1 \), he cannot, in general, simultaneously increase both \( s_1 \) and \( s_2 \). Thus, if by a small change of his own strategy, player 1 adds a small value \( \theta_1 \) (either positive or negative) to his own survival probability during an encounter and \( \theta_2 \) (positive or negative) to his companion's, then this change will be for his long-term advantage if and only if

\[
\theta_1 \frac{\partial \psi}{\partial s_1} + \theta_2 \frac{\partial \psi}{\partial s_2} > 0
\]

(3.15)

Thus, a pure strategy \( \Gamma \) (say, a measurable set \( \Gamma \), over which the individual is willing to cooperate) is a strict ESS if when both players choose \( \Gamma \), the reverse of (3.15) holds with respect to any small change made by player 1, either in the direction of more cooperation, less cooperation, or cooperation at different situations.

The analysis of the exact set of all possible strict ESSs is complicated since it involves problems of measure and integration. It can be shown, though (Eshel & Weinshall, 1988), that an infinite set of strict ESSs exists, each determining cooperation on a wider set of situations. But even the minimal Ess always determines cooperation on a non-degenerate set of encounters of the prisoner’s dilemma type.

Repeated game with a random encounter rule -

The general case of dependence between death events

In the general case, death of one player at a given encounter is not independent of the death of the other. Thus, full information about the survival probabilities of the two players in a given

encounter is given by the 2 x 2 matrix of quadruplets \((x_{11}^k, x_{10}^k, x_{01}^k, x_0^k)\), \(k = 1, 2\) where

- \( x_{11}^k \) is the probability that both survive
- \( x_{10}^k \) is the probability that only the first one survives
- \( x_{01}^k \) is the probability that only the second one survives
- \( x_0^k \) is the probability that none survive

all under the assumption that the first player takes the \( i \)-th and the second takes the \( j \)-th alternative decision of cooperate non-cooperate.

Denote by \( s_{11}, s_{10}, s_{01} \) and \( s_0 \), the appropriate one encounter survival probabilities, and let \( s_1 = s_{11} + s_{10}, s_2 = s_{01} + s_0 \) (the survival probabilities of the first player and the second player, respectively, in a single encounter). These values are calculated as functions of \( \Gamma_1 \) and \( \Gamma_2 \) very much in the same way as shown in (3.6) and survival until the end of the supergame is easily shown (very much as in (3.9)-(3.12) to be

\[
V(\Gamma_1, \Gamma_2) = \sum_{i=0}^{\infty} p^i q s_{11}^i = \left[ \frac{1}{1 - \sum_{i=0}^{\infty} p^i q s_{11}^i} \right] \frac{s_{10}}{1 - s_{11}} \sum_{i=0}^{\infty} p^i q s_{11}^i
\]

\[
= \frac{q}{1 - p s_{11}} + \left[ \frac{1}{1 - p s_{11}} \right] \frac{s_{10}}{1 - s_{11}} \frac{q}{1 - \lambda q}
\]

\[
= \frac{q}{1 - \lambda} \left[ 1 + p s_{11} + s_{10} \lambda \right]
\]

\[
= \frac{q}{1 - \lambda} \left( 1 + p \psi(s_{11}, s_{10}) \right)
\]

(4.1)
where
\[ \varphi(s_1, s_{10}) = \frac{s_1 - \lambda}{1 - s_{11}} = \frac{s_1 - \lambda}{1 - s_{11} - s_{10} - s_{110}} = \psi(\Gamma_1, \Gamma_2) \]  
(4.2)
and we assume that player 1, by changing \( \Gamma_1 \), is seeking to increase this function. As before, \( \Gamma \) is a strict ESS if, when chosen by both players, player 1, by exclusively changing his own strategy, can change \( s_1 \) and \( s_{10} \) (or \( s_1 \) and \( s_{10} \)) only in such a way that the value \( \varphi(s_1, s_{10}) \) will decrease.

Analysis of the entire set of possible ESSs is similar to that done for the case of independent death events and, except for the case \( s_{11} = 0 \) (in which at each encounter, only one of the two players can survive), there is always a non-degenerate set of encounters of the prisoner’s dilemma type (i.e., a set with a positive probability) on which there is full cooperation under any of the ESS strategies.

However, it can be shown that as the correlation between death events increases, the range of cooperation increases. In the extreme case of negative correlation, on the other hand, where \( s_{11} = 0 \), and hence \( s_1 = s_2 = s_{10} \), (4.2) becomes
\[ \varphi(s_1, s_{10}) = s_1 - \lambda \]  
(4.3)
independently on \( s_{10} \) or \( s_{11} \), hence, in this case, player 1 is absolutely indifferent to the fate of player 2, and no cooperation can evolve.

Discussion: Partnership altruism versus reciprocal altruism and kinship altruism

Partnership altruism (help your partner now in order that he will be able to help you in the future, when worthwhile to him) is to be distinguished from reciprocal altruism (help your partner in order that he will remember and return in kind for kindness; Trivers, 1971). In this work we suggest a basic model for partnership altruism based on physical survival. The same model can be applied to survival in a wider sense, say staying in business, avoiding bankruptcy, keeping academic position, etc., and the idea of partnership altruism can easily be extended beyond that of preventing full destruction of the partner. In many cases it is in one’s own interest to keep one’s partner rich, healthy, or powerful in order that he will be more efficient in providing help in the future.

Unlike reciprocal altruism, partnership altruism does not assume gratefulness, a rather problematic component in the attempt to explain the evolution of altruism on the basis of reciprocity. Indeed, being ungrateful, when in your own interest, may be as tempting as defecating in encounters of the prisoner’s dilemma type. In fact, we see that the evolution of partnership altruism does not even require memory, and the model suggested may thus be applicable to the evolution of apparently altruistic cooperation among non-relatives, observed in some populations of very primitive organisms (e.g., see Eshel & Cohen, 1976). Yet, as we see in this discussion, memory can be useful in enabling the partners to switch from one ESS level of cooperation to a higher one for the benefit of both. The existence of infinitely many such ESSs has been demonstrated in the analysis of the model and it is not very surprising intuitively. Indeed, as a higher level of cooperation is established between two partners, each becomes more valuable for the other from the point of view of future perspective help and this in turn can perpetuate the new, high level of cooperation (up to some upper limit). (See for comparison Eshel & Motro, 1981).

This possibility makes it advantageous for any partner to let his altruistic action be known to the other (a factor which, unlike in reciprocal altruism, is not a necessary prerequisite for help). At the same time, it is as advantageous for the recipient of the altruistic help to notice and remember the level of altruistic cooperation of his partner as to adjust himself to the appropriate ESS level and to stick to the highest level obtained, to his own benefit.

But is such an adjustment to the partner’s level of altruistic behavior not equivalent, from a behavioral point of view, to gratefulness? Indeed, a rational partner may calculate the future advantage of adjusting his behavior to that of his partner, but more likely, as the same behavior can result from a sincere feeling of gratefulness, natural selection will favor the evolution of a tendency to feel grateful and act accordingly up to some level. The model of partnership altruism, thus, does not mean to replace that of reciprocal altruism but, instead, to provide a complementary explanation to its evolution. Yet factors of partnership altruism can be distinguished in practice from those of pure reciprocity by the possible tendency of a participant to help (or in most real cases, not to harm too much when in one’s immediate advantage) another participant even when that action (or inaction) is certain not to be known to the other.
Finally, developing a model for partnership and reciprocity, we do not suggest here that these are the only or even the main factors in the evolution of human (and animal) altruism. Early human population structure seems likely to be most favorable for the evolution of altruism due to kin selection (Hamilton, 1964) or weak group selection, say neighbor-effect (Eshel, 1972). Moreover, as it has been shown (Axelrod & Hamilton, 1981), a tendency towards altruistic cooperation, originally evolving on the basis of kin selection, may be only a first step towards the establishment of stable altruistic cooperation (in this case, tit-for-tat) among non-relatives.

However, as indicated by the result of this work, partnership altruism can have crucial aspects that make it qualitatively different from kinship altruism. These aspects can be revealed (in daily life or, suggestedly, in laboratory experiments) only if we observe the individual’s behavior under different types of encounters.

The qualitative difference between partnership and kinship altruism stems from the fact that in the case of partnership, one is interested in the survival of one’s companion (or in improving his status) only as long as the partnership can be continued, first of all, as long as he himself is alive, or have a substantial chance to remain alive, together with his partner. Pure kinship altruism, on the other hand, is aimed to benefit the relative as potential carrier of one’s own genes (Hamilton, 1964) and as such, is likely to be endowed (e.g., in terms of life insurance, last will, etc.) even after death. (For the fact that any kinship altruism must include a factor of partnership altruism, though; see Eshel & Moto, 1981. Indeed, if a relative is supposed to help you because he is a relative, then once he is dead you are bound to lose more than his genes. You lose a potential helper as well as perhaps a competitor, a factor that must affect kinship.)

To illustrate the difference between kinship and partnership altruism, let us return to the imaginary example of two companions encountering a lion. Each of them may either stay and fight or escape. Assuming the following symmetric matrix of individual survival:

<table>
<thead>
<tr>
<th></th>
<th>Fight (0.5;0.5)</th>
<th>Escape (0.0;0.7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fight</td>
<td>(0.7;0)</td>
<td>(0.2;0.2)</td>
</tr>
</tbody>
</table>

First, assume the case of a one-shot encounter between relatives. Indeed, by sticking to one’s companion (as an alternative to exclusive escape), one decreases one’s chance of survival by 0.2 while increasing his relative’s chance by 0.5. Thus, the inclusive fitness, accrued by this action, is 0.5r - 0.2, where r is the coefficient of relatedness between them. Cooperation is stable if and only if r > 0.4. Thus, in the case of kinship altruism, the only information needed for the decision whether to cooperate or to defect is given by the matrix of individual survival probabilities.

Suppose, on the other hand, that the two companions are non-relatives, but they know they still have to find their way out of the forest, and the chance of each of them surviving alone (after escaping the lion) is θ times smaller than in the presence of his companion. Does the matrix of individual survival (together with the value of θ) give us sufficient information to make a rational decision in this case? The answer is no. One must resort to additional information about the correlation between the death events of the two companions. To illustrate this, assume the two extreme situations, both fitting well to the information given above in the matrix of individual survival.

**Case I.** Fighting together, the partners have a 0.5 chance to chase the lion away. If not, they both die.

In this case, sticking to your partner and fighting together, you have a 0.5p chance to escape safely from the forest (p being the chance of escaping the forest with a partner). Escaping the lion you have a chance of 0.7θp, hence cooperation is beneficial if and only if θ < 5/7

**Case II.** Fighting together, the lion will kill just one of the two.

This still agrees with the figure of 0.5 individual chance of survival given by the matrix. But whatever θ may be, escaping the lion is always advantageous because, in this case, if you survive, you remain alone anyhow, and by escaping you at least increase your first survival probability.

In the same way it might be shown, more generally, that if the correlation between the death events in fighting the lions is -1 < R < 1, then cooperation is advantageous in the long run if and only if

\[
\frac{1 + R}{4} \cdot \frac{1 - R}{4} θ > 0.70 \quad \text{or} \quad θ < \frac{1 - R}{4.8 - R}
\]
and correlation between the partner's survival events becomes a crucial factor in partnership altruism while it does not affect kinship altruism.

Keeping this in mind, the notorious saying "after me, the deluge" may not necessarily reflect utmost selfishness. Instead, it may reflect a change of attitude from a paternal approach of a king as a "father of the nation" (fitting the model of Axelrod & Hamilton, 1981), to a more modern approach towards a political leader as a senior partner whose best interest may (though, unfortunately as we know, may not) lie in the welfare of his disciples, as long as he remains in power and indeed stays alive, but not after his death.

Footnotes

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References


