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On Sums of Units

By

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Abstract. It is shown that if R is a finitely generated integral domain of zero characteristic, then for every n there exist elements of R which are not sums of at most n units. This implies in particular to rings of integers in finite extensions of the rationals. On the other hand there are many infinite algebraic extensions of the rationals in which every integer is a sum of two units.

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1. An integral domain R is called (see [1]) n-good, if every element of R can be written as a sum of n units, i.e., invertible elements of R, and it is called ω-good, if it is not n-good for any finite n, but each of its elements is a sum of units. It has been recently proved by Ashrafi and Vámos ([1]) that the ring of integers of a quadratic number field is not n-good for any n, and the same holds in the case of cubic number fields having a negative discriminant and cyclotomic fields $\mathbb{Q}(\zeta_{2^N})$ for every $N \geqslant 1$. In this note we shall establish this result for all rings of integers of algebraic number fields. Actually we shall prove this in a more general situation:

Theorem 1. *If R is a finitely generated integral domain of zero characteristic, then there is no integer n such that every element of R is a sum of at most n units.*

We shall obtain this as a simple corollary of van der Waerden's theorem and a finiteness result concerning unit equations. The use of Szemerédi's theorem will then lead to the assertion that for $n = 1, 2, \ldots$ the set of positive rational integers which are sums of at most n units in a fixed algebraic number field has zero density.

2. Let R be a domain, and denote by $U(R)$ its group of units. An equality of the form

$$c = u_1 + u_2 + \cdots + u_m \tag{1}$$

with a non-zero $c \in R$ and $u_i \in U(R)$ will be called *proper*, if the sum on the right hand-side does not contain vanishing sub-sums.

We shall use the following two known results:

Lemma 2. (See [2]) *If R is a finitely generated integral domain of zero characteristic, then for every $n = 2, 3, \dots$ there exists a constant $B_n(R)$ such that for every non-zero $c \in R$ and $m = 2, 3, \dots, n$ the Eq.* (1) *has at most $B_n(R)$ proper unit solutions $(u_1, u_2, \dots, u_m)$.*

The second result which will be used is a version of van der Waerden's theorem:

Lemma 3. *Let r, s be fixed positive integers and let S be an arithmetic progression of rational integers of length N. If N is sufficiently large, and S is a union of r sets, then at least one of them contains an arithmetic progression of s terms.*

Proof. Let a, b be positive integers, let

$$S = \{a + md\colon \quad m = 1, 2, \dots, N\} = \bigcup_{i=1}^{r} S_i,$$

and for $i = 1, 2, \dots, r$ put $A_i = \{m\colon\ a + md \in S_i\}$.

Van der Waerden's theorem ([7]) gives a constant $W(r, s)$ such that if N exceeds $W(r, s)$ (for the best known effective upper bound for $W(r, s)$ see [4]), then for a certain i the set A_i contains an arithmetic progression of s terms, and so does S_i. □

3. Theorem 1 is a direct consequence of the following lemma:

Lemma 4. *If R is a finitely generated integral domain of zero characteristic and $n \geqslant 1$ is an integer, then there exists a constant $A_n(R)$ such that every arithmetic progression in R having more than $A_n(R)$ elements contains an element which is not a sum of n units.*

Proof. We apply induction in n. Let first $n = 1$, $\delta \neq 0$ and let $a_j = a_0 + (j-1)\delta$ $(j = 1, 2, \dots, N)$ be an arithmetic progression consisting of units of R. Since for $j = 0, 1, \dots, N-1$ we have $a_{j+1} - a_j = \delta$, hence the equation $x + y = \delta$ has at least N unit solutions, thus $N \leqslant B_2(R)$. It follows that we may put $A_1(R) = B_2(R)$.

Assume now that the assertion of the lemma holds for a certain $n \geqslant 1$, denote for non-zero $\delta \in R$ by $\Omega_0(\delta)$ the set of all units u which appear in a proper equality of the form

$$\delta = u_1 + u_2 + \cdots + u_m$$

with $m = 1, 2, \dots, 2n+2$ in which $u_1, u_2, \dots, u_m$ are units, and put

$$\Omega(\delta) = \{\pm u\colon \quad u \in \Omega_0(\delta)\} = \{x_1, x_2, \dots, x_M\}.$$

Lemma 2 implies that M is bounded by a number, depending only on R and n.

Consider now a finite arithmetic progression $a_j = a_0 + (j-1)\delta \in R$ $(j = 1, 2, \dots, N)$, each term of which is a sum of $n+1$ units. We have to show that N does not exceed a bound, depending only on n and R. For $n = 1, 2, \dots, N$ we have

$$a_j = \sum_{r=1}^{n+1} u_{r,j},$$

with $u_{r,j} \in U(R)$. This implies

$$\delta = a_{j+1} - a_j = \sum_{r=1}^{n+1} u_{r,j+1} - \sum_{r=1}^{n+1} u_{r,j} \tag{2}$$

for $j = 1, 2, \ldots, N$. The right hand-side of these equalities is non-zero, and so after possible cancellations we obtain a proper equality, hence for each j at least one of the units appearing in (2) lies in $\Omega(\delta)$. Without restricting the generality we may assume that for every j either $u_{1,j}$ or $u_{1,j+1}$ lies in $\Omega(\delta)$.

If now we put for $t = 1, 2, \ldots, M$

$$X_t = \{1 \leqslant j \leqslant N: \quad u_{1,j} = x_t\}$$

and

$$Y_t = \{1 \leqslant j \leqslant N: \quad u_{1,j+1} = x_t\},$$

then

$$\{1, 2, \ldots, N\} = \bigcup_{t=1}^{M} X_t \cup \bigcup_{t=1}^{M} Y_t.$$

By Lemma 3 at least one of the sets $X_1, \ldots, X_M, Y_1, \ldots, Y_M$ contains an arithmetic progression P of length $T > A_n(R)$, provided N is sufficiently large. Without restricting the generality we may assume that the set X_1 has this property. Let h be the difference of P. Now write $P = \{n_1, n_2, \ldots, n_T\}$ with $n_i = i_0 + (i-1)h$, and put $b_i = a_{n_i} - x_1$ $(i = 1, 2, \ldots, T)$. Then

$$b_i = a_{i_0+(i-1)h} - x_1 = (a_0 + (i_0 - 1)\delta - x_1) + (i-1)h\delta,$$

hence $b_1, b_2, \ldots, b_T$ is an arithmetic progression of length exceeding $A_n(R)$ in contradiction to the induction hypothesis. □

Theorem 1 is an immediate consequence of the lemma.

4. We point out some simple corollaries:

Corollary 5. *A finitely generated integral domain of zero characteristic cannot be n-good for any n. This holds, in particular, for the ring of integers of every number field of finite degree.*

Proof. This follows directly from the theorem.

Corollary 6. *Let K be a finite extension of the rationals, and for each positive integer n and $x \geqslant 1$ denote by $N_n(x)$ the number of positive rational integers $m \leqslant x$, which are sums of at most n units of K. Then*

$$\lim_{x\to\infty} \frac{N_n(x)}{x} = 0. \tag{3}$$

Proof. If (3) would fail, then according to Szemerédi's theorem (see [6]), there would exist arbitrarily long progressions of positive rational integers $m \leqslant x$, which are sums of at most n units. Lemma 3 implies that in this case for a certain integer

$l \leqslant n$ there would exist arbitrarily long progressions of elements which are sums of l units, contrary to Lemma 4. $\square$

Corollary 7. *Let $n \geqslant 1$ be a given integer, let $p_1, p_2, \ldots, p_r$ be fixed primes, and put*

$$A = \{\pm p_1^{a_1} \cdots p_r^{a_r} : \ a_i \in \mathbb{Z}\}.$$

Then the set of positive integers which are sums of at most n elements of A has density zero.

Proof. This follows from Lemma 4 applied to the ring $\mathbb{Z}\left[\frac{1}{M}\right]$ with $M = \prod_{i=1}^{r} p_i$, and Szemerédi's theorem. $\square$

5. Let K be an algebraic extension of the rationals and S a finite set of non-archimedean prime divisors of K. For each $\mathfrak{p} \in S$ choose a Henselian closure $K_\mathfrak{p}$ of K at $\mathfrak{p}$, and let $\bar{K}_\mathfrak{p}$ be its residue field. Denote by $\tilde{K}$ the algebraic closure of K, and let $\mathrm{Gal}(K)$ be the absolute Galois group of K. Let $K_{\mathrm{tot},S}$ be the maximal Galois extension of K in which all primes of S split completely, i.e.,

$$K_{\mathrm{tot},S} = \bigcap_{\mathfrak{p} \in S} \bigcap_{\tau \in \mathrm{Gal}(K)} K_\mathfrak{p}^\tau,$$

where $K_\mathfrak{p}^\tau = \tau(K_\mathfrak{p})$.

Let m be a positive integer. For each $\boldsymbol{\sigma} = (\sigma_1, \ldots, \sigma_m) \in \mathrm{Gal}(K)^m$ let $\tilde{K}(\boldsymbol{\sigma})$ be the fixed field of $\sigma_1, \ldots, \sigma_m$ in $\tilde{K}$, and put

$$\tilde{K}_{\mathrm{tot},S}(\boldsymbol{\sigma}) = \tilde{K}(\boldsymbol{\sigma}) \cap K_{\mathrm{tot},S}.$$

In the following we shall use the expression "for almost all $\boldsymbol{\sigma} \in \mathrm{Gal}(K)^m$" in the sense of the Haar measure of the profinite group $\mathrm{Gal}(K)^m$ (see [3], Chap. 18).

Finally, for every algebraic extension M/K denote by O_M the ring of integers of M.

Theorem 8. *Assume that for each $\mathfrak{p} \in S$ one has $|\bar{K}_\mathfrak{p}| \geqslant 3$, and let $n \geqslant 2$ be a rational integer. Then for almost all $\boldsymbol{\sigma} \in \mathrm{Gal}(K)^m$ every element of the ring R of integers of the field $\tilde{K}_{\mathrm{tot},S}(\boldsymbol{\sigma})$ is a sum of n units.*

Proof. Our argument is based on the following assertion, which is a combination of Corollary 1.9, Theorem 1.5 and Remark 1.3 (c) of [5]:

Lemma 9. (Local-global principle) *The following statement holds for almost all $\boldsymbol{\sigma} \in \mathrm{Gal}(K)^m$: Let V be an affine absolutely irreducible smooth variety over the field $M = \tilde{K}_{\mathrm{tot},S}(\boldsymbol{\sigma})$. Suppose that $V(O_{\tilde{K}}) \neq \emptyset$ and for each prime $\mathfrak{q}$ of M lying over S one has $V(O_{M_\mathfrak{q}}) \neq \emptyset$. Then $V(O_M)$ is non-empty.*

It suffices to establish the theorem in the case $n = 2$, as the general case follows from the observation that if a ring R is 2-good, then it is n-good for all $n \geqslant 2$. Indeed, if this holds for a certain n and $r \in R$, then

$$r = u_1 + u_2 + \cdots + u_n$$

with units $u_1, \ldots, u_n$, and if we write $u_n = u + v$ with units u, v, then $r = u_1 + \cdots + u_n + u + v$ is a sum of $n+1$ units.

Let $a \in R$. Consider the affine variety V defined over R by the following equations:

$$X_1 + X_2 = a, \qquad X_1 Y_1 = 1, \qquad X_2 Y_2 = 1. \tag{1}$$

The variety V has an $O_{\tilde{K}}$-rational point. Indeed, let x_1, x_2 be roots of the polynomial $X^2 - aX + 1$. Then $x_1, x_2 \in O_{\tilde{K}}$, $x_1 + x_2 = a$ and $x_1 x_2 = 1$. Thus x_1, x_2 are units, and if we put $y_i = x_i^{-1}$ $(i = 1, 2)$, then $(x_1, x_2, y_1, y_2) \in V(O_{\tilde{K}})$.

Now let $\mathfrak{P}$ be a prime of M over S. Then $\bar{M}_{\mathfrak{P}}$ is a finite field of at least three elements, so there exists $x_1 \in O_{M_{\mathfrak{p}}}$ such that the elements $x_1, a - x_1, 0$ are distinct modulo $\mathfrak{P}$. If $x_2 = a - x_1$, then x_1, x_2 are units of $O_{M_{\mathfrak{P}}}$, and $a = x_1 + x_2$. As before, put $y_i = x_i^{-1}$ $(i = 1, 2)$ to get $(x_1, x_2, y_1, y_2) \in V(O_{\tilde{K}})$.

Since V is smooth and absolutely irreducible, Lemma 4 is applicable to R, so there exist $(x_1, x_2, y_1, y_2) \in V(R)$. Therefore x_1, x_2 are units of R, and $a = x_1 + x_2$, as desired. □

6. We conclude with three open questions related to sums of units:

Problem A. *Give a criterion for an algebraic extension K of the rationals to have the property that O_K is α-good for some $\alpha \in \{1, 2, \ldots, \omega\}$.*

This has been solved for quadratic number fields in [1] (Theorems 7 and 8).

It follows from the Kronecker-Weber theorem that the maximal Abelian extension $\mathbb{Q}^{ab}$ of the rationals has this property. Indeed, if $a \in \mathbb{Q}^{ab}$ is an integer, then it lies in a suitable cyclotomic field $K_n = \mathbb{Q}(\zeta_n)$, ζ_n being a primitive n-th root of unity, and since the ring of integers of K_n equals $\mathbb{Z}[\zeta_n]$ we can write

$$a = c_0 + c_1 \zeta_n + \cdots + c_r \zeta_n^r,$$

with $c_j \in \mathbb{Z}$, and $r = [K_n : \mathbb{Q}] - 1$. Therefore a is a sum of at most $\sum_{j=0}^{r} |c_j|$ of units. We do not know, whether the ring of integers of $\mathbb{Q}^{ab}$ is n-good for a finite n.

One sees easily that the ring of integers of every algebraic extension of the rationals which is closed under quadratic extensions is 2-good. Indeed, if α is an algebraic integer, and u, v are roots of the polynomial $f(X) = X^2 - \alpha X - 1$, then u, v are both units, and $u + v = \alpha$. In particular the ring of all algebraic integers is 2-good, and since the discriminant of f equals $\alpha^2 + 4$, hence is positive for real α, the same applies to the ring of all real algebraic integers.

The criterion given in [1] provides examples of quadratic fields K for which O_K is not ω-good. However every such field has a finite extension L such that the ring O_L is ω-good. This leads to the next question:

Problem B. *Is it true that each number field has a finite extension L such that O_L is ω-good?*

Problem C. *Let K be an algebraic number field. Obtain an asymptotical formula for the number $N_k(x)$ of positive rational integers $n \leqslant x$ which are sums of at most k units of the field K.*

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