Syllabus for the course "Smooth Functions"

First semester, 2010/11, course number: 0366-4764

Fix a dimension $n \geq 2$ and a degree of smoothness m. Suppose $E \subset \mathbb{R}^n$ is a closed set (perhaps a large finite set), and $f : E \to \mathbb{R}$. How can you extend the function f to a smooth function on the entire $\mathbb{R}^n$ of minimal C^m norm? How can you compute the order of magnitude of this minimal C^m norm? What can you say on the extending function?

In the second part of the course we will discuss a recent breakthrough obtained by Charlies Fefferman regarding the above questions. This has lead to a deeper understanding of the nature of C^m smooth functions.

In the first part of the course we will learn some preliminary material, regarding C^m-smooth functions in $\mathbb{R}^n$. My general plan for the first half of the course is roughly as follows:

1. Sard's lemma
2. Whitney's extension theorem for jets.
3. Algorithmic aspects of Whitney's theorem.
4. One-dimension, difference quotients.
5. Loseless extensions: Kirszbraun theorem and related facts.
6. Fefferman's gentle partitions of unity.
7. Brudnyi-Shvartsman finiteness principle, Lipschitz selections.
8. The $C^{1,1}$ case of Fefferman's proof.

The course assumes certain mathematical maturity in classical analysis. The formal prerequisites for the course are: Calculus 3 (0366-2141), Real Analysis (0366-2106), and Introduction to Computer Science for Mathematicians (0366-1106). The first two courses are completely essential, the third one is less important.

We will meet on Mondays, 15 - 18, Ornstein 102. There will not be class in the first week on November.