Algebraic Methods in Combinatorics 0366-4933

Michael Krivelevich

Spring Semester 2013

Homework 2

Due: Apr. 24, 2013

1. Prove (algebraically!) that every set P of n points in the plane, not all are on one line, determines at least n distinct lines (a line is determined by P if it passes through a pair of points in P).

2. Let $\mathcal{F}_1, \mathcal{F}_2 \subseteq \binom{[n]}{k}$ be k-uniform families. Let L_1, L_2 be two disjoint sets of integers such that $\mathcal{F}_i$ is L_i-intersecting, $i = 1, 2$. Prove that $|\mathcal{F}_1| \cdot |\mathcal{F}_2| \leq \binom{n}{k}$.

3. Let p be a prime number, and let $L \subseteq Z_p$ be a subset of s residues. Suppose that $A_1, \ldots, A_m$ and $B_1, \ldots, B_m$ are two families of subsets of $[n]$ satisfying:

1. $|A_i \cap B_i| \pmod p \notin L$ for all $1 \leq i \leq m$;
2. $|A_j \cap B_i| \pmod p \in L$ for all $1 \leq j < i \leq m$.

Prove that $m \leq \binom{n}{\leq s} = \sum_{i=0}^{s} \binom{n}{i}$.

4. Show that for intifinely many values of n one can *explicitly* construct a 2-coloring of the edges of the complete bipartite graph K_{n^2,n^2} without a monochromatic copy of $K_{n+1,n+1}$.

5. For an integer $k \geq 2$, give an explicit coloring of the edges of the complete graph on $n = \binom{t}{2k-1}$ vertices in k colors, not containing a monochromatic copy of K_{t+1}.