1. Prove (algebraically!) that every set $P$ of $n$ points in the plane, not all are on one line, determines at least $n$ distinct lines (a line is determined by $P$ if it passes through a pair of points in $P$).

2. Let $\mathcal{F}_1, \mathcal{F}_2 \subseteq \binom{[n]}{k}$ be $k$-uniform families. Let $L_1, L_2$ be two disjoint sets of integers such that $\mathcal{F}_i$ is $L_i$-intersecting, $i = 1, 2$. Prove that $|\mathcal{F}_1| \cdot |\mathcal{F}_2| \leq \binom{n}{k}$.

3. Let $p$ be a prime number, and let $L \subseteq \mathbb{Z}_p$ be a subset of $s$ residues. Suppose that $A_1, \ldots, A_m$ and $B_1, \ldots, B_m$ are two families of subsets of $[n]$ satisfying:

1. $|A_i \cap B_i| \pmod{p} \notin L$ for all $1 \leq i \leq m$;

2. $|A_j \cap B_i| \pmod{p} \in L$ for all $1 \leq j < i \leq m$.

Prove that $m \leq \binom{n}{\leq s} = \sum_{i=0}^{s} \binom{n}{i}$.

4. Show that for infinitely many values of $n$ one can explicitly construct a 2-coloring of the edges of the complete bipartite graph $K_{n^2,n^2}$ without a monochromatic copy of $K_{n+1,n+1}$.

5. For an integer $k \geq 2$, give an explicit coloring of the edges of the complete graph on $n = \binom{t}{2k-1}$ vertices in $k$ colors, not containing a monochromatic copy of $K_{t+1}$. 