1. $n$ people meet at a party. They discover that each pair of the party attendees has an odd number of common acquaintances at the party. Prove that $n$ is odd.

*Hint.* Show first that each person knows an even number of people at the party.

2. Show that no graph has eigenvalue $-1/2$.

3. Let $G$ be a bipartite graph. Prove that if 0 is not an eigenvalue of the adjacency matrix $A(G)$, then $G$ has a perfect matching.

4. For a graph $G = (V, E)$, denote by $L(G)$ its line graph. (The vertices of $L(G)$ are the edges of $G$, two edges $e_1 \neq e_2 \in E(G)$ are connected by an edge in $L(G)$ iff $e_1$ and $e_2$ share a vertex.) Prove that all eigenvalues of $L(G)$ are at least $-2$.

5. Let $G$ be a graph on $n$ vertices. Suppose that the all-one matrix $J_n$ is a linear combination of the powers of $A(G)$. Show that $G$ is connected and regular.