Algebraic Methods in Combinatorics 0366-4933

Michael Krivelevich

Spring Semester 2013

Homework 4

Due: June 19, 2013 - **SHARP!**

1. n people meet at a party. They discover that each pair of the party attendees has an odd number of common acquaintances at the party. Prove that n is odd.
Hint. Show first that each person knows an even number of people at the party.

2. Show that no graph has eigenvalue $-1/2$.

3. Let G be a bipartite graph. Prove that if 0 is not an eigenvalue of the adjacency matrix $A(G)$, then G has a perfect matching.

4. For a graph $G = (V, E)$, denote by $L(G)$ its line graph. (The vertices of $L(G)$ are the edges of G, two edges $e_1 \neq e_2 \in E(G)$ are connected by an edge in $L(G)$ iff e_1 and e_2 share a vertex.) Prove that all eigevalues of $L(G)$ are at least -2.

5. Let G be a graph on n vertices. Suppose that the all-one matrix J_n is a linear combination of the powers of $A(G)$. Show that G is connected and regular.