

Graph Theory 0366-3267  
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Homework Assignment No. 4  
Due: Jan. 25, 2012

1. Show that if

$$3 \binom{n}{k} 3^{-\binom{k}{2}} < 1$$

then there is a coloring of the edges of the complete graph on  $n$  vertices by 3 colors with no monochromatic clique of size  $k$ .

2. Let  $A$  be a set of  $3m$  points in the Euclidean plane, and suppose that the distance between any two of these points is smaller than  $\sqrt{2}$ . Prove that the number of pairs  $P, Q$  of points of  $A$  so that the distance between  $P$  and  $Q$  is at least 1 does not exceed  $3m^2$ .

3. Show that if the edges of a graph  $G$  can be covered by two trees then its chromatic number is at most 4.

4. Let  $G$  be a simple graph with maximum degree 7 containing no clique of size 4. Prove that the chromatic number of  $G$  is at most 6.

Hint: Show first that one can delete from  $G$  a bipartite graph leaving each degree in what's left at most 3.

5. Let  $G$  be a graph with chromatic number  $\chi(G) = 11$  and with no cycle of length at most 20. Show that the number of vertices of  $G$  exceeds the world population (which is close to 7,000,000,000).

6. Let  $G$  be a 2-connected, simple 5-regular planar graph drawn in the plane so that every face contains the same number of edges. What is the number of vertices of  $G$ ? Prove your claim.