Graph Theory 0366-3267
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Homework 1
Due: Nov. 28, 2012

1. Prove that every simple graph with $n \geq 7$ vertices and at least $5n - 14$ edges contains a subgraph with minimum degree at least 6.

2. Prove that every graph $G = (V, E)$ with $|E| = m$ edges has a bipartition $V = V_1 \cup V_2$ such that the number of edges of G crossing between V_1 and V_2 is at least $m/2$.

3. (a) Let G be a graph with all degrees at least three. Prove that G contains a cycle with a chord.

(b) Let G be a graph on $n \geq 4$ vertices with $2n - 3$ edges. Prove that G contains a cycle with a chord.

4. Let $G = (V, E)$ be a connected graph with an even number of vertices. Prove that G contains a spanning subgraph $G' = (V, F)$ with all degrees odd.

5. Prove that for every integer $k > 0$ there exists integer n such that every tree T on n vertices contains at least k leaves or a path P of length k with all vertices of degree exactly 2 in T.

6. Let G be graph on $n \geq 4$ vertices with $2n - 2$ edges. Prove that G has two cycles of equal length.

7. Prove that every graph G with minimal degree d contains every tree on $d + 1$ vertices as a subgraph.

8. Compute the number of spanning trees in the complete bipartite graph $K_{m,n}$.