Graph Theory 0366-3267

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Homework 3
Due: Jan. 9, 2013

1. Let $G = (A \cup B, E)$ be a bipartite graph with a perfect matching, in which every vertex $a \in A$ has degree at least k in G, for some integer $k \geq 1$. Prove that G contains at least $k!$ perfect matchings. (**Hint:** recall the two cases in the proof of Hall's theorem...)

2. Prove the so called defect version of Hall's theorem: Let $G = (A \cup B, E)$ be a bipartite graph. Define

$$\delta(A) = \max_{S \subseteq A}(|S| - |N(S)|) \ .$$

Prove that $\nu(G) = |A| - \delta(A)$, where $\nu(G)$ is the matching number of G.

3. Let G be a connected graph with an even number of edges. Prove, using Tutte's Theorem, that the set of edges of G can be partitioned into pairwise disjoint pairs, where each pair forms a path of length 2.

4. Prove that every k-regular $(k-1)$-edge-connected graph on an even number of vertices has a perfect matching.

6. Let G be a graph on n vertices. Prove: $\chi(G) \cdot \chi(\bar{G}) \geq n$.

7. Let G be a graph in which every pair of odd cycles has a common vertex. Prove that $\chi(G) \leq 5$.

8. Let $G = (V, E)$ be a graph of maximum degree Δ. Prove that there is a $(\Delta + 1)$-edge coloring of G so that each color is used either $\left\lfloor \frac{|E(G)|}{\Delta+1} \right\rfloor$ or $\left\lceil \frac{|E(G)|}{\Delta+1} \right\rceil$ times.