

Graph Theory 0366-3267

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Homework 4
Due: Jan. 16, 2013

1. Prove: $R(3,4)=9$.
2. Let m be given. Show that if n is large enough, then every n -by- n 0,1-matrix has a principal submatrix of size m in which all elements above the diagonal are the same, and all elements below the diagonal are the same.
3. Prove that for every positive integer k there exists an integer $n = n(k)$ such that every k -coloring of all the subsets of $\{1 \dots, n\}$ contains two disjoint non-empty sets A, B such that A, B and $A \cup B$ have the same color.
4. Prove that for every tree T and every integer g there exists a graph G without cycles of length up to g and such that every two-coloring of the edges of G contains a monochromatic copy of T .
5. Let $G = (V, E)$ be a graph with 1000 vertices and 250001 edges. Prove: G contains two triangles sharing an edge.
6. 30 people need to place a call to each other using their cellular phones (one call per each pair). A cellular phone company gets 1 shekel for each call between two people at distance between 800 and 1000 meters. The company is allowed to locate the people as it wishes so as to maximize its profit. What is the maximum possible profit of the company?