Graph Theory 0366-3267

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Homework 1
Due: Nov. 17, 2013

1. Prove that every simple graph with $n \geq 7$ vertices and at least $5n - 14$ edges contains a subgraph with minimum degree at least 6.

2. Prove that every graph $G = (V, E)$ with $|E| = m$ edges has a bipartition $V = V_1 \cup V_2$ such that the number of edges of G crossing between V_1 and V_2 is at least $m/2$.

3. **(a)** Let G be a graph with all degrees at least three. Prove that G contains a cycle with a chord.

(b) Let G be a graph on $n \geq 4$ vertices with $2n-3$ edges. Prove that G contains a cycle with a chord.

4. Let $G = (V, E)$ be a graph containing two edge disjoint spanning trees. Prove that G contains a spanning subgraph $H = (V, F)$, such that H is connected and all of its vertex degrees are even.

5. Let G be graph on $n \geq 4$ vertices with $2n - 2$ edges. Prove that G has two cycles of equal length.

6. Let $0 < d_1 \leq d_2 \leq \ldots \leq d_n$ be integers. Prove that there exists a tree with degrees $d_1, \ldots, d_n$ if and only if

$$d_1 + \ldots + d_n = 2n - 2\,.$$

7. Let $d \geq 0$ be an integer. Prove that every graph G with minimum degree d contains every tree on $d + 1$ vertices as a subgraph.

8. Compute the number of spanning trees in the complete bipartite graph $K_{m,n}$.