Graph Theory 0366-3267

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Homework 2

Due: Dec. 8, 2013

1. Prove that in a connected graph G every two paths of maximum length share a vertex.

2. Let Q_k be the k-dimensional cube defined as follows: $V(Q_k) = \{0,1\}^k$, $(\mathbf{x} = (x_1, \ldots, x_k), \mathbf{y} = (y_1, \ldots, y_k)) \in E(Q_k)$ iff $\mathbf{x}$ and $\mathbf{y}$ differ in exactly one coordinate. Prove: $\kappa(Q_k) = \kappa'(Q_k) = k$.

3. Let $k \geq 2$. Prove that every k-connected graph on at least $2k$ vertices contains a cycle of length at least $2k$.

4. Let G be a graph in which every pair of vertices has an odd number of common neighbors. Prove that G is Eulerian.

5. A mouth eats its way through a $3 \times 3 \times 3$ cube of cheese by eating all the $1 \times 1 \times 1$ subcubes. If it starts at a corner subcube and always moves on to an adjacent subcube (sharing a face of area 1), can it do this and eat the center subcube last?

6. Let G be a connected graph on n vertices with $\delta(G) = d$. Prove that G contains a path of length $\min\{2d, n-1\}$.

7. Let $t(n, H_n)$ be the maximum number of edges in a graph G on n vertices, not containing a Hamilton cycle H_n. Prove: $t(n, H_n) = \binom{n-1}{2} + 1$. (You need to prove both lower and upper bounds for $t(n, H_n)$.)