1. Prove that in a connected graph $G$ every two paths of maximum length share a vertex.

2. Let $Q_k$ be the $k$-dimensional cube defined as follows: $V(Q_k) = \{0, 1\}^k$, $(x = (x_1, \ldots, x_k), y = (y_1, \ldots, y_k)) \in E(Q_k)$ iff $x$ and $y$ differ in exactly one coordinate. Prove: $\kappa(Q_k) = \kappa'(Q_k) = k$.

3. Let $k \geq 2$. Prove that every $k$-connected graph on at least $2k$ vertices contains a cycle of length at least $2k$.

4. Let $G$ be a graph in which every pair of vertices has an odd number of common neighbors. Prove that $G$ is Eulerian.

5. A mouth eats its way through a $3 \times 3 \times 3$ cube of cheese by eating all the $1 \times 1 \times 1$ subcubes. If it starts at a corner subcube and always moves on to an adjacent subcube (sharing a face of area 1), can it do this and eat the center subcube last?

6. Let $G$ be a connected graph on $n$ vertices with $\delta(G) = d$. Prove that $G$ contains a path of length $\min\{2d, n - 1\}$.

7. Let $t(n, H_n)$ be the maximum number of edges in a graph $G$ on $n$ vertices, not containing a Hamilton cycle $H_n$. Prove: $t(n, H_n) = \binom{n-1}{2} + 1$. (You need to prove both lower and upper bounds for $t(n, H_n)$.)