Graph Theory 0366-3267

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Homework 3
Due: Jan. 11, 2015

1. Let $A = (a_{i,j})$ be an n by n real matrix, where $n > 1$, $a_{i,j} \geq 0$ for all i, j and the sum of elements in each row of A and the sum of elements in each column of A is exactly 1. Show that there is a permutation σ of $1, 2, \ldots, n$ so that $a_{i,\sigma(i)} > \frac{1}{n^2}$ for all $1 \leq i \leq n$.

2. Prove the so called defect version of Hall's theorem: Let $G = (A \cup B, E)$ be a bipartite graph. Define

$$\delta(A) = \max_{S \subseteq A}(|S| - |N(S)|) \ .$$

Prove that $\nu(G) = |A| - \delta(A)$, where $\nu(G)$ is the matching number of G.

3. Let $G = (A \cup B)$ be a bipartite graph, in which for every edge $e = (a, b) \in E$ with $a \in A, b \in B$ one has $d(a) \geq d(b)$, and also $d(a) > 0$ for every $a \in A$. Show that G has a matching saturating A.

4. Let G be a connected graph with an even number of edges. Prove, using Tutte's Theorem, that the set of edges of G can be partitioned into pairwise disjoint pairs, where each pair forms a path of length 2.

5. Let G be a graph on n vertices. Prove: $\chi(G) \cdot \chi(\bar{G}) \geq n$.

6. Let G be a graph in which every pair of odd cycles has a common vertex. Show that $\chi(G) \leq 5$.

7. Let G be a union of k forests. Prove that $\chi(G) \leq 2k$.

8. Let G be a graph in which every edge belongs to at most k cycles. Show that $\chi(G) \leq k+2$.